Metamaterials: Definitions, limitations, chirality and bi-anisotropy

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Outline

- Definition of metamaterial
- Effective-medium model
- Spatial dispersion
  - Chirality
  - Artificial magnetism
- Reciprocal bi-anisotropic materials
- Limitations on material parameters

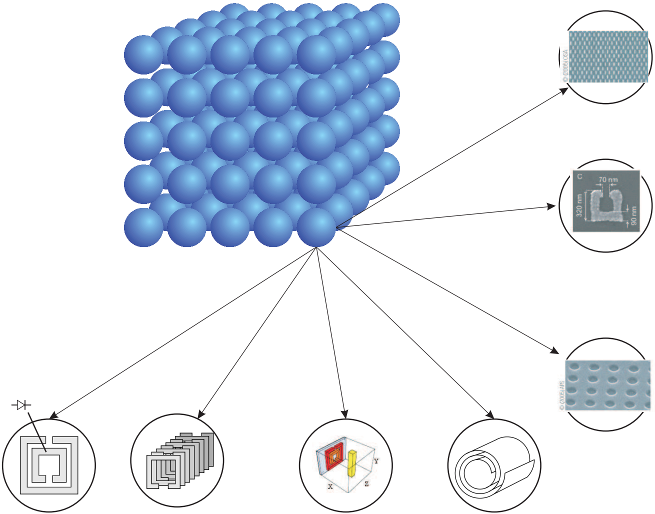
Definition: *Metamaterial*

Meta- denotes position behind, after, or beyond, and also something of a higher or second-order kind. . .

Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties.

More precisely, properties that cannot be achieved at the atomic or molecular level are achieved through the electromagnetic properties of “particles" formed at levels much higher than the atomic level but whose dimensions are small compared to the wavelength of operation.

Metamaterial concept



Physically sound (local) parameters

$$\mathbf{D} = \epsilon \mathbf{E}, \qquad \mathbf{B} = \mu \mathbf{H}$$

- Model a linear response of the medium
- Satisfy the causality requirement (Kramers-Kronig relations)
- Satisfy the passivity requirement (II law of thermodynamics)
- Are independent of the spatial distribution of fields excited in the material sample
- Are independent of the geometrical size and shape of the sample

for all linear passive media in thermodynamically equilibrium states.

Difficulties

in effective-parameter description of metamaterials:

- Period is not very small compared with the wavelength $\Rightarrow$ Spatial dispersion is not negligible
- Number of layers/inclusions is not very large $\Rightarrow$ Surface effect is not negligible
- Particles are resonant $\Rightarrow$ Spatial dispersion can be significant not only near lattice resonances
- Inclusions have both electric and magnetic responses $\Rightarrow$ Effective permeability is supposed to model both spatial dispersion near lattice (Bragg) resonance and magnetic polarization due to individual inclusions

Macroscopic Maxwell equations

Volume averaging:

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}, \qquad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0^{-1} \nabla \times \mathbf{B} = j\omega\epsilon_0 \mathbf{E} + \langle \mathbf{J} \rangle , \qquad \epsilon_0 \nabla \cdot \mathbf{E} = \langle \rho \rangle$$

$$\langle \rho \rangle = \rho^{\text{ind}} + \rho^{\text{ext}}, \qquad \langle \mathbf{J} \rangle = \mathbf{J}^{\text{ind}} + \mathbf{J}^{\text{ext}}$$

Conservation of induced charge:

$$\nabla \cdot \mathbf{J}^{\text{ind}} + j\omega\rho^{\text{ind}} = 0$$

Spatial dispersion

Let us define induction vectors as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \frac{\mathbf{J}^{\mathrm{ind}}}{j\omega}, \qquad \mathbf{H} = \mu_0^{-1} \mathbf{B}$$

Then

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}, \quad \nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}^{\mathrm{ext}}, \quad \nabla \cdot \mathbf{D} = \rho^{\mathrm{ext}}, \quad \nabla \cdot \mathbf{B} = 0$$

Polarization is induced by *electric* field, thus

$$\mathbf{J}^{\mathrm{ind}}(\mathbf{r}) = \int_V \overline{\overline{K}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\, dV'$$

No natural magnetic fraction: no need to include response directly on $\mathbf{B}$.

Strong and weak spatial dispersion

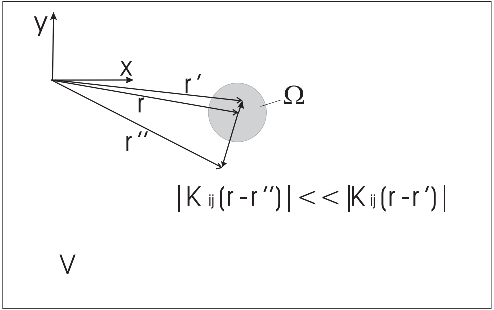


Figure: $J_i = \int_V K_{ij}(\mathbf{r}-\mathbf{r}')E_j(\mathbf{r}')\,dV'$. Generally $K_{ij}|_{|\mathbf{r}-\mathbf{r}'|\to\infty} \to 0$.

- $\Omega > \lambda/2$ – strong spatial dispersion (SD)
- $\Omega \ll \lambda$ – weak SD
- Ω is negligibly small – no SD

Weak spatial dispersion

$$\mathbf{J}^{\text{ind}}(\mathbf{r}) = \int_V \overline{\overline{K}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')\,dV' \approx \int_\Omega \overline{\overline{K}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')\,dV', \quad k\Omega < 1$$

$$\mathbf{E}(\mathbf{r}') = \mathbf{E}(\mathbf{r}) + (\partial_\alpha \mathbf{E})\Big|_{\mathbf{r}}(r'_\alpha - r_\alpha) + \frac{1}{2}(\partial_\beta\partial_\alpha \mathbf{E})\Big|_{\mathbf{r}}(r'_\alpha - r_\alpha)(r'_\beta - r_\beta) + \cdots$$

Taking into account spatial derivatives up to the second order:

$$J_i^{\text{ind}} = j\omega\left[a_{ij}E_j + a_{ijk}(\nabla_k E_j) + a_{ijkl}(\nabla_l \nabla_k E_j)\right]$$

Second-order spatial dispersion

Constitutive relations:

$$D_i = (\epsilon_0 \delta_{ij} + a_{ij}) E_j + a_{ijk} (\nabla_k E_j) + a_{ijkl} (\nabla_l \nabla_k E_j)$$

$$\mathbf{H} = \mu_0^{-1} \mathbf{B}$$

(J.W. Gibbs, 1882)

These definitions imply unusual boundary conditions. We want to transform the fields to let them satisfy the usual ones.

$$\mathbf{D} = \epsilon\mathbf{E} + \alpha\nabla\times E + \beta\nabla\nabla\cdot\mathbf{E} + \gamma\nabla\times\nabla\times\mathbf{E}$$

$$\mathbf{H} = \mu_0^{-1}\mathbf{B}$$

The Maxwell equations are invariant with respect to transformation

$$\mathbf{D}' = \mathbf{D} + \nabla\times\mathbf{Q}, \quad \mathbf{H}' = \mathbf{H} + j\omega\mathbf{Q}$$

Indeed,

$$\nabla\times\mathbf{H} = j\omega\mathbf{D} \Rightarrow$$

$$\nabla\times\mathbf{H}' - j\omega\nabla\times\mathbf{Q} = j\omega\mathbf{D}' - j\omega\nabla\times\mathbf{Q} \Rightarrow$$

$$\nabla\times\mathbf{H}' = j\omega\mathbf{D}'$$

First-order spatial dispersion

$$\mathbf{D} = \epsilon \mathbf{E} + \alpha \nabla \times \mathbf{E}, \qquad \mathbf{H} = \mu_0^{-1} \mathbf{B}$$

Transformation with $\mathbf{Q} = -\frac{\alpha}{2}\mathbf{E}$. The constitutive relations transform as

$$\mathbf{D}' = \epsilon \mathbf{E} + \frac{\alpha}{2} \nabla \times \mathbf{E} = \epsilon \mathbf{E} - j\omega \frac{\alpha}{2} \mathbf{B}$$

$$\mathbf{H}' = \mathbf{H} = \mu_0^{-1} \mathbf{B} - j\omega \frac{\alpha}{2} \mathbf{E}$$

Material relations for isotropic chiral media:

$$\mathbf{D}' = \epsilon \mathbf{E} - j\xi \mathbf{B}, \qquad \mathbf{H}' = \mu_0^{-1} \mathbf{B} - j\xi \mathbf{E}$$

(the chirality parameter $\xi = \omega\alpha/2$)

$$\mathbf{D} = \epsilon\mathbf{E} - j\xi\mathbf{B} + \beta\nabla\nabla\cdot\mathbf{E} + \gamma\nabla\times(\nabla\times\mathbf{E})$$

$$\mathbf{H} = \mu_0^{-1}\mathbf{B} - j\xi\mathbf{E}$$

Because $\nabla\times\mathbf{E} = -j\omega\mathbf{B}$, we can try to transform the relations in such a way that the curl would disappear.

Transforming again, with $\mathbf{Q} = -\gamma\nabla\times\mathbf{E} = j\omega\gamma\mathbf{B}$, we get

$$\mathbf{D} = \epsilon\mathbf{E} - j\xi\mathbf{B} + \beta\nabla\nabla\cdot\mathbf{E}, \qquad \mathbf{H} = \mu^{-1}\mathbf{B} - j\xi\mathbf{E}$$

$$\mu = \frac{\mu_0}{1 - \omega^2\mu_0\gamma}$$

Artificial "magnetism"...

Conclusions about effective medium models

Be careful!

- Effective medium parameters (permittivity, permeability,...) model volume-averaged properties of materials
- Chirality is a first-order effect of spatial dispersion
- Magnetic response of materials without naturally-magnetic components is a second-order spatial dispersion effect
- In the general case there are other spatial-dispersion effects of the same order that are not modelled by the effective permeability
- There are some specific structures for which the permittivity/permeability model is appropriate, within certain limits