Double negative (DNG) metamaterials

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Lecture plan

- Double-negative metamaterials
  - Negative parameters
  - Physical limitations
- Plane waves in DNG media
  - Backward waves
  - Negative refraction
  - Plasmons
  - Perfect lens

Negative material parameters

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} = -\epsilon_0 |\epsilon_r| \mathbf{E}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} = -\mu_0 |\mu_r| \mathbf{H}$$

where the relative material parameters ϵ_r, μ_r are real and negative

More generally, both $\mathrm{Re}\{\epsilon_r\} < 0$ and $\mathrm{Re}\{\mu_r\} < 0$.

materials with negative parameters
backward-wave media
double negative (DNG) media
materials with negative refraction index (NRI)
left-handed materials
Veselago media

First DNG/Veselago material

R.A. Shelby, et al., *Science*, vol. 292, pp. 77-79, 2001.

Quasi-static model of wire media

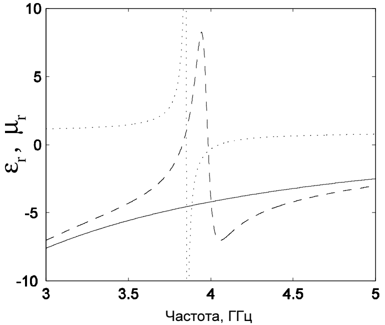
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \text{where} \quad \mathbf{P} = \frac{\mathbf{J}}{j\omega} = \mathbf{z}_0 \frac{I}{j\omega a^2} = -\mathbf{z}_0 \frac{E_z}{\omega^2 a^2 L}$$

Material relation:

$$D_z = \left(\epsilon_0 - \frac{1}{\omega^2 a^2 L} \right) E_z$$

$$L = \frac{\mu_0}{2\pi} \log \frac{a^2}{4 r_0 (a - r_0)}$$

Wire medium + an artificial magnetic



Negative permeability background of wire medium ⇒ positive(!) permittivity of wire medium

Limitations on material parameters

For low-loss materials:

$$\frac{d\epsilon(\omega)}{d\omega} > 0, \qquad \frac{d\epsilon(\omega)}{d\omega} > \frac{2(\epsilon_0 - \epsilon)}{\omega}$$

From here:

$$\frac{d(\omega\epsilon(\omega))}{d\omega} > \epsilon_0, \qquad \frac{d(\omega\mu(\omega))}{d\omega} > \mu_0$$

Also,

$$\frac{d(\omega\epsilon(\omega))}{d\omega} > 2\epsilon_0 - \epsilon(\omega)$$

Energy density

Considering metamaterials with negligible losses (in some frequency ranges):

$$w = \frac{1}{2}\left.\frac{d(\omega\epsilon(\omega))}{d\omega}\right|_{\omega=\omega_0}|E|^2 + \frac{1}{2}\left.\frac{d(\omega\mu(\omega))}{d\omega}\right|_{\omega=\omega_0}|H|^2$$

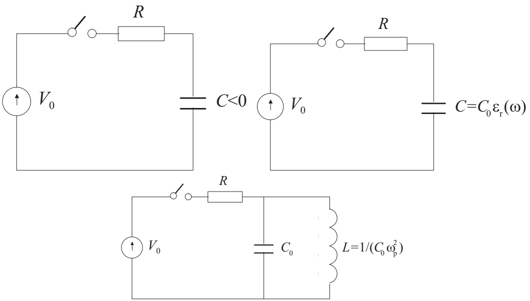
Assume that ϵ and μ are independent from the frequency (near ω_0):

$$w = \frac{1}{2}\epsilon(\omega_0)|E|^2 + \frac{1}{2}\mu(\omega_0)|H|^2$$

But $w > 0$ in passive media!

Conclusion: It is not possible to neglect dispersion if the material parameters are negative.

Filled capacitor



$$i(t) = V_0 \omega C \frac{\cos(\omega t) + \omega RC \sin(\omega t) - e^{-\frac{t}{RC}}}{1 + \omega^2 R^2 C^2}$$ — instability!

$$\epsilon(\omega) = \epsilon_0 \epsilon_r = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \Rightarrow$$

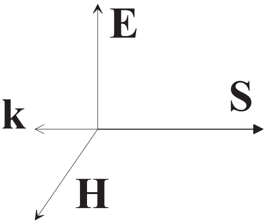
$$Z = \frac{1}{j\omega C} = \frac{1}{j\omega C_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right)} = \frac{1}{j\omega C_0 + \frac{\omega_p^2 C_0}{j\omega}}$$

Plane waves

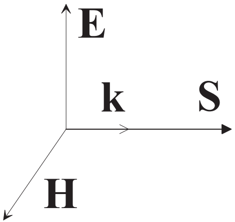
Backward waves:

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}, \qquad \mathbf{k} \times \mathbf{H} = -\omega\epsilon\mathbf{E}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \frac{E^2}{\omega\mu}\mathbf{k} = \frac{H^2}{\omega\epsilon}\mathbf{k}$$

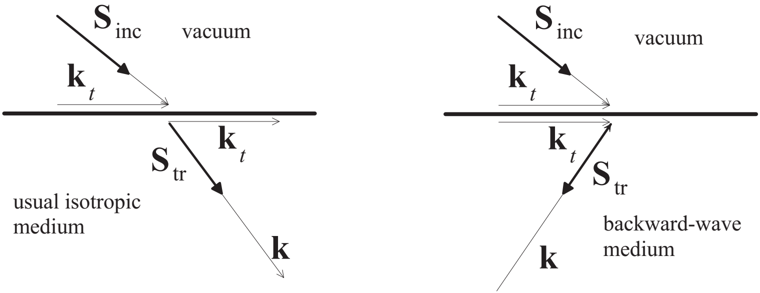


Plane wave in a Veselago medium



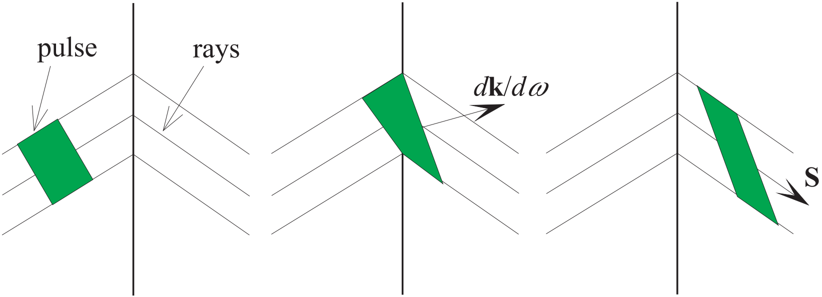
Plane wave in a usual isotropic medium

Negative refraction



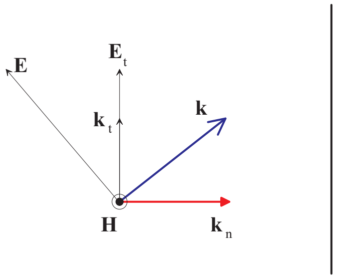
Negative refraction of beams, cont.

S. Maslovski, rejected submission to *Phys. Rev. Lett.*, July 2002.



Propagation of a space-time modulated pulse: increasing moments of time, from left to right.

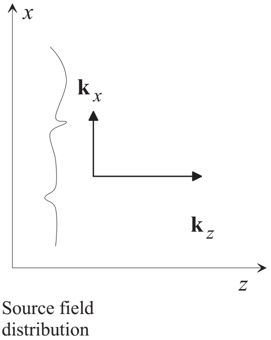
Plane-wave incidence



$$\eta = \frac{E_t}{H} = \frac{E\cos\theta}{H} = \sqrt{\frac{\mu}{\epsilon}}\frac{k_n}{k} = \frac{k_n}{\omega\epsilon}$$

$$k_n = \sqrt{k^2 - k_t^2}$$

Evanescent and propagating modes



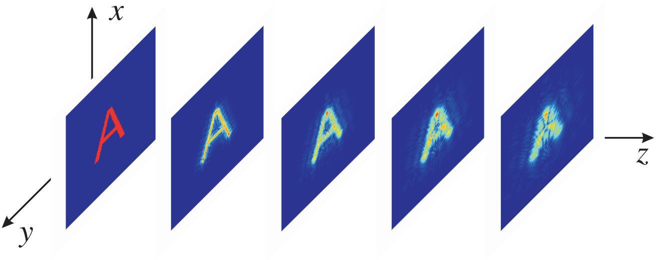
$$k_x^2 + k_y^2 + k_z^2 = k^2$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = \sqrt{k^2 - k_t^2}, \quad k_t^2 = k_x^2 + k_y^2.$$

Assuming no losses:

- $k_t < k \Rightarrow k_z$ is real, wave **propagates**
- $k_t > k \Rightarrow k_z$ is imaginary, wave decays (**evanescent** wave)

Image transfer



$$\mathbf{E}(x,y,z) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \mathbf{E}(k_x,k_y)\, e^{-j(k_x x + k_y y \pm \sqrt{k^2 - k_x^2 - k_y^2}\, z)}\, dk_x\, dk_y$$

$k_\mathrm{t}^2 = k_x^2 + k_y^2$, $k_\mathrm{t} < k$: propagating, $k_\mathrm{t} > k$: evanescent.

Consider an interface between free space and a Veselago material (TM waves).

$$R = \frac{\eta - \eta_0}{\eta + \eta_0}, \qquad T = \frac{2\eta}{\eta + \eta_0}$$

$$\eta = \frac{k_\mathrm{n}}{\omega\epsilon}, \qquad k_n = \sqrt{k^2 - k_t^2}$$

$$\eta_0 = \frac{k_\mathrm{n}}{\omega\epsilon_0}, \qquad k_n = \sqrt{k_0^2 - k_t^2}$$

In a Veselago medium $\epsilon < 0$ and $\mu < 0$, and $k_\mathrm{n} < 0$ (backward-wave medium).
If $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$, we have $k = -k_0$, $k^2 = k_0^2$, $\eta = \eta_0$, and

$$R = 0, \qquad T = 1$$

For evanescent waves

$$k_0 = \sqrt{k_0^2 - k_t^2} = -j\alpha, \quad k = \sqrt{k_0^2 - k_t^2} = -j\alpha_0, \qquad \alpha > 0, \quad \alpha_0 > 0$$

$$\eta = \frac{-j\alpha}{\omega\epsilon}, \qquad \eta_0 = \frac{-j\alpha_0}{\omega\epsilon_0}$$

.

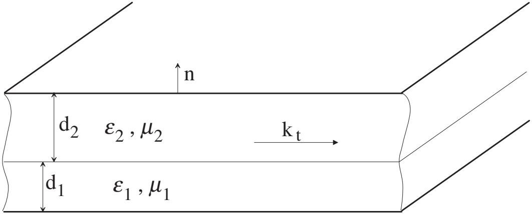
When $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$, we have purely imaginary wave impedances such that $\eta = -\eta_0$ for all k_t, and a resonance occurs:

$$T, R \rightarrow \infty$$

.

Surface mode (surface plasmon).

Two-layer system



Dispersion equation:

$$\frac{k_{\mathrm{n}1}}{\epsilon_1}\tan k_{\mathrm{n}1}d_1+\frac{k_{\mathrm{n}2}}{\epsilon_2}\tan k_{\mathrm{n}2}d_2=0,\qquad \text{TM modes}$$

$$\frac{\mu_1}{k_{\mathrm{n}1}}\tan k_{\mathrm{n}1}d_1+\frac{\mu_2}{k_{\mathrm{n}2}}\tan k_{\mathrm{n}2}d_2=0,\qquad \text{TE modes}$$

$$k_{\mathrm{n}1,2}=\sqrt{k_{1,2}^2-k_t^2}$$

Phase compensation

Let $k_t = 0$ and consider standing waves between two metal boundaries

Eigenvalue equation

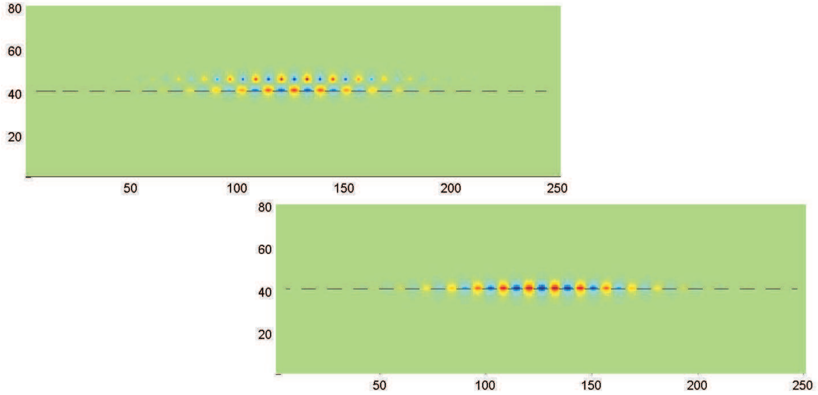
$$\frac{\mu_1}{k_1} \tan k_1 d_1 + \frac{\mu_2}{k_2} \tan k_2 d_2 = 0$$

Thin layers:

$$\mu_1 d_1 + \mu_2 d_2 = 0$$

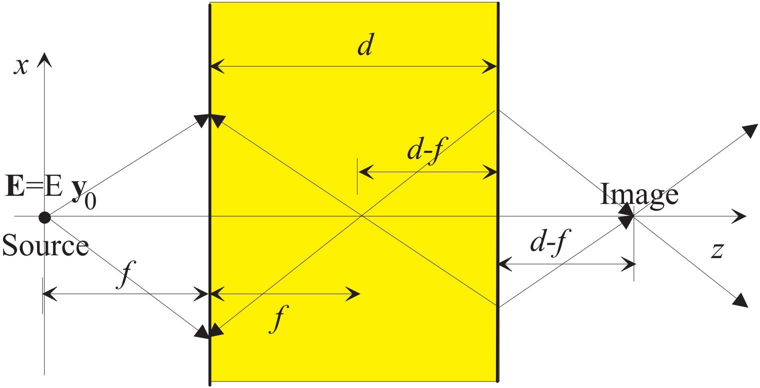
N. Engheta, An idea for thin subwavelength cavity resonators using metamaterials with negative permittivity and permeability, *IEEE Antennas and Propagation Lett.*, vol. 1, no. 1, pp. 10-13, 2002.

Memory "device"



S.A. Tretyakov, S.I. Maslovski, I.S. Nefedov, M.K. Kärkkäinen, Evanescent modes stored in cavity resonators with backward-wave slabs, *Microwave and Optical Technology Letters*, vol. 38, no. 2, pp. 153-157, 2003.

Perfect lens



V. Veselago, 1967 (propagating waves); J. Pendry, 2000 (all modes).

Reflection and transmission

Consider an EVANESCENT incident plane wave

$$\mathbf{E} = E\mathbf{y}_0 e^{-jk_x x - \alpha_0 z}, \qquad H_x = -\frac{\alpha_0}{j\omega\mu_0} E_y$$

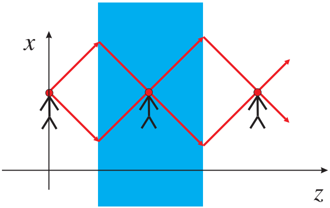
where $\alpha_0 = \sqrt{k_x^2 - k_0^2} > 0$

$$R = \frac{\frac{1}{2}\left(\frac{\alpha_0\mu}{\alpha\mu_0} - \frac{\alpha\mu_0}{\alpha_0\mu}\right)\sinh\alpha d}{\cosh\alpha d + \frac{1}{2}\left(\frac{\alpha_0\mu}{\alpha\mu_0} + \frac{\alpha\mu_0}{\alpha_0\mu}\right)\sinh\alpha d}$$

$$T = \frac{1}{\cosh\alpha d + \frac{1}{2}\left(\frac{\alpha_0\mu}{\alpha\mu_0} + \frac{\alpha\mu_0}{\alpha_0\mu}\right)\sinh\alpha d}$$

For $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$ we get $R = 0, \quad T = e^{\alpha d}$

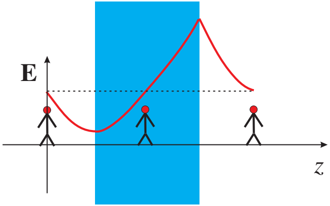
Two phenomena:



Propagating modes
— negative refraction

$$\epsilon_r = -1, \qquad \mu_r = -1$$

$$n = \sqrt{\epsilon_r \mu_r} = -1$$



Evanescent modes
— plasmon resonance

$$R_{\text{half}} \to \infty$$

How it all works?

Let the lens be excited by a current line. The incident wave "spatial spectrum"

$$\int_{-\infty}^{\infty} H_0^{(2)}\left[k\sqrt{(x^2+z^2)}\right] e^{-jk_x x}\,dx = \frac{2}{\sqrt{k^2-k_x^2}} e^{-j\sqrt{k^2-k_x^2}|z|}$$

Source just at the first interface. On the other side of the lens the propagating waves become

$$\frac{2}{\sqrt{k^2-k_x^2}} e^{+j\sqrt{k^2-k_x^2}\,d}, \qquad k_x < k$$

But the evanescent part of the spectrum transforms like

$$\frac{2}{\sqrt{k^2-k_x^2}} e^{\sqrt{k_x^2-k^2}\,d}, \qquad k_x > k$$

Next, to the focus:

$$\frac{2}{\sqrt{k^2-k_x^2}} e^{+j\sqrt{k^2-k_x^2}\,d} e^{-j\sqrt{k^2-k_x^2}\,d} = \frac{2}{\sqrt{k^2-k_x^2}}, \qquad k_x < k$$

$$\frac{2}{\sqrt{k^2-k_x^2}} e^{\sqrt{k_x^2-k^2}\,d} e^{-\sqrt{k_x^2-k^2}\,d} = \frac{2}{\sqrt{k^2-k_x^2}}, \qquad k_x > k$$

Limitations:

- Reflections from the lens perimeter
- Discrete structure of the lens material
- Losses
- Dispersion
- ...

Problem!

The integral for the field on the back side of the lens diverges

$$\int_{k}^{\infty} \frac{2}{\sqrt{k^2 - k_x^2}} e^{\sqrt{k_x^2 - k^2}\, d} \, e^{j k_x x} \, dk_x = \infty$$

Solution: When k_t grows, at some point the effective medium model becomes not applicable.