Basics of electromagnetic wave interaction with composite materials: polarization responses, dispersion, bianisotropics

Training Workshop for new FP7 projects on metamaterials

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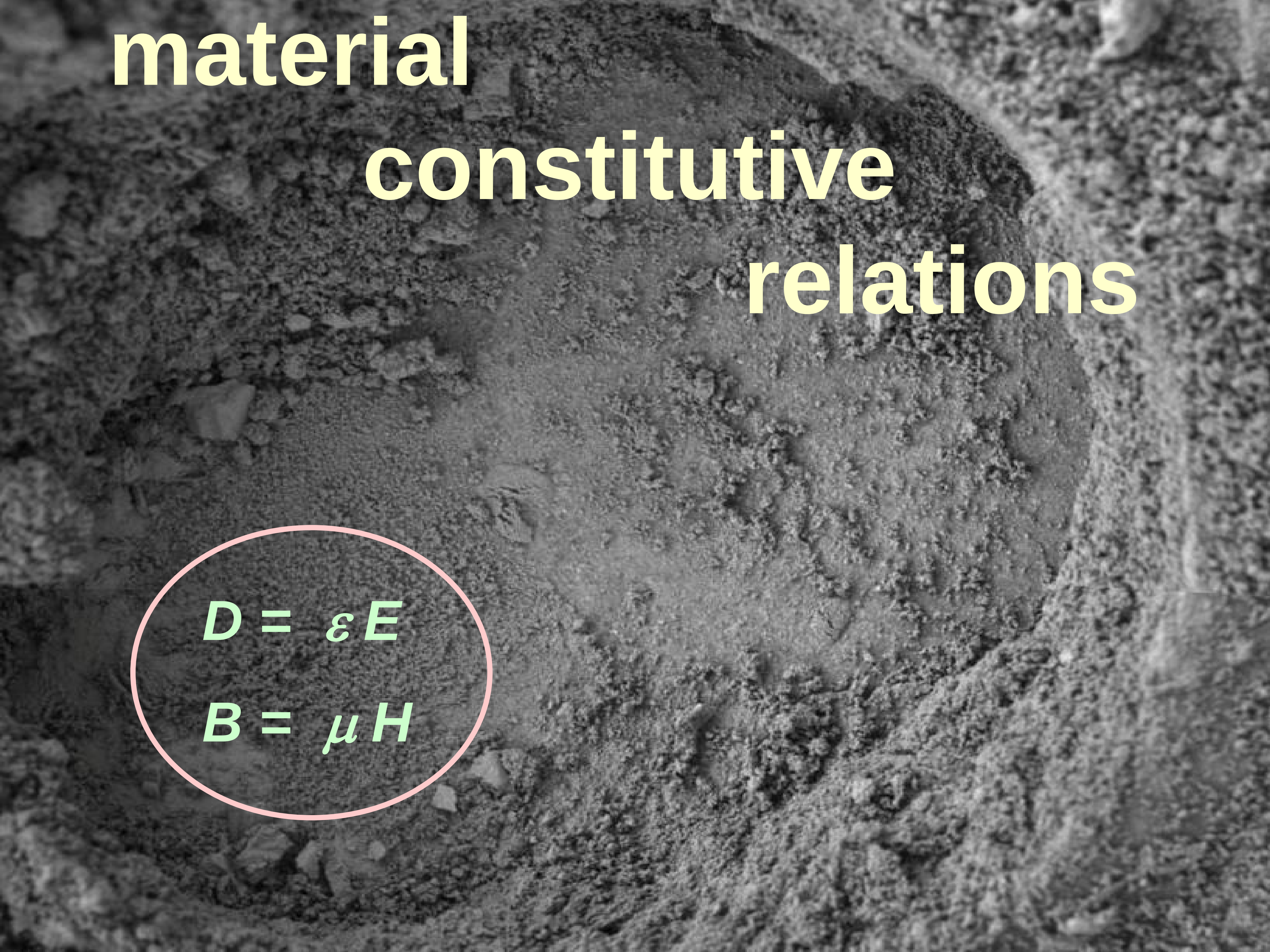
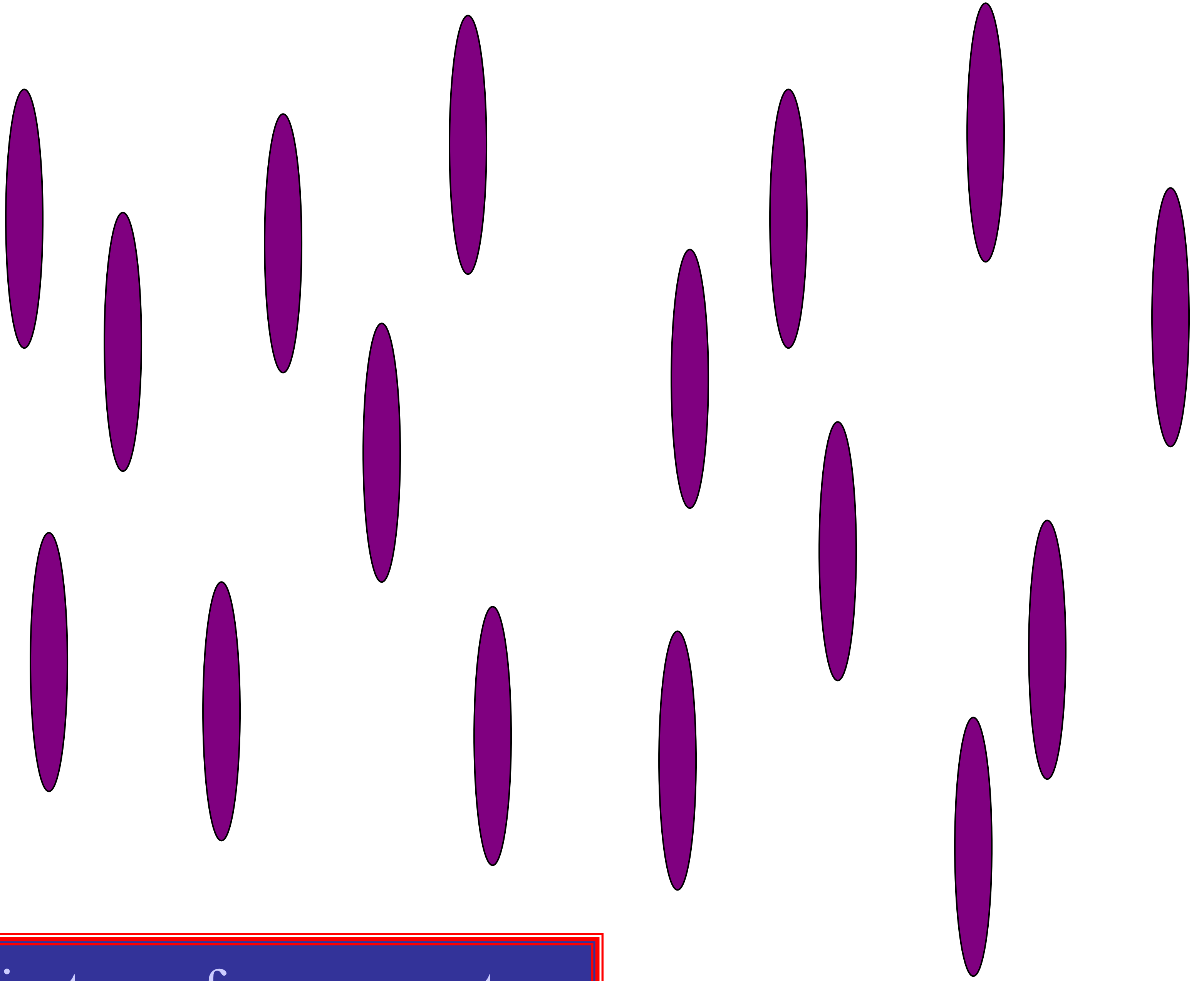
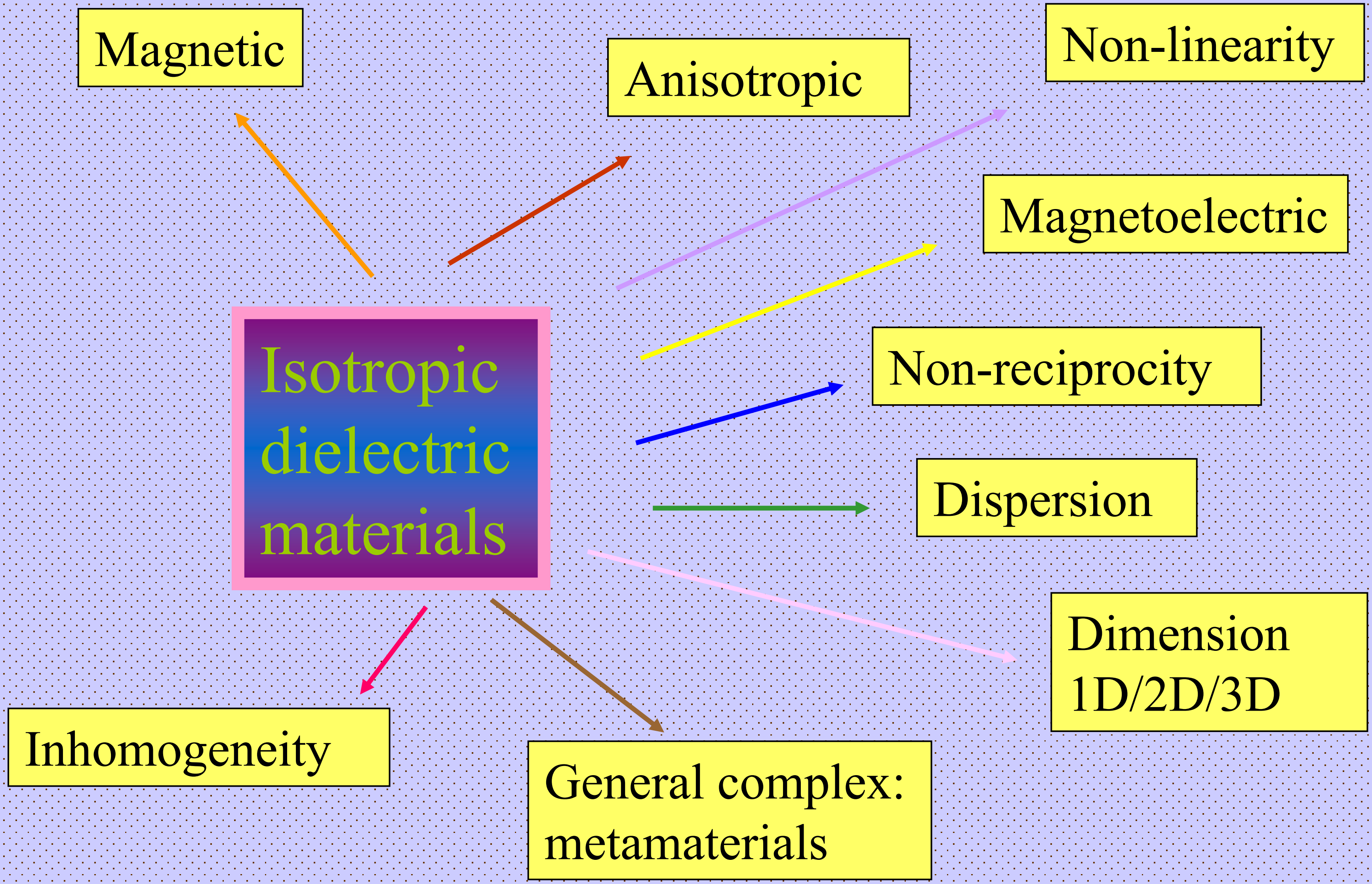
Topics to be discussed

- geometry affects matter
- chirality
- anisotropy
- bi-anisotropy
- reaction and reciprocity
- dissipation
- temporal dispersion
- Lorentz, Drude, Debye
- Kramers–Kronig & their global character
- mixing principles


material
constitutive
relations
$D = \varepsilon E$
$B = \mu H$



Anisotropy from geometry


Magnetic
Anisotropic
Non-linearity
Magnetoelectric
Isotropic dielectric materials
Non-reciprocity
Dispersion
Dimension 1D/2D/3D
Inhomogeneity
General complex: metamaterials

Polarization response

$$D = \varepsilon_0 E + n\,p + \ldots$$

$$p = \alpha E$$

$$p = \underline{\underline{\delta}} : EE$$

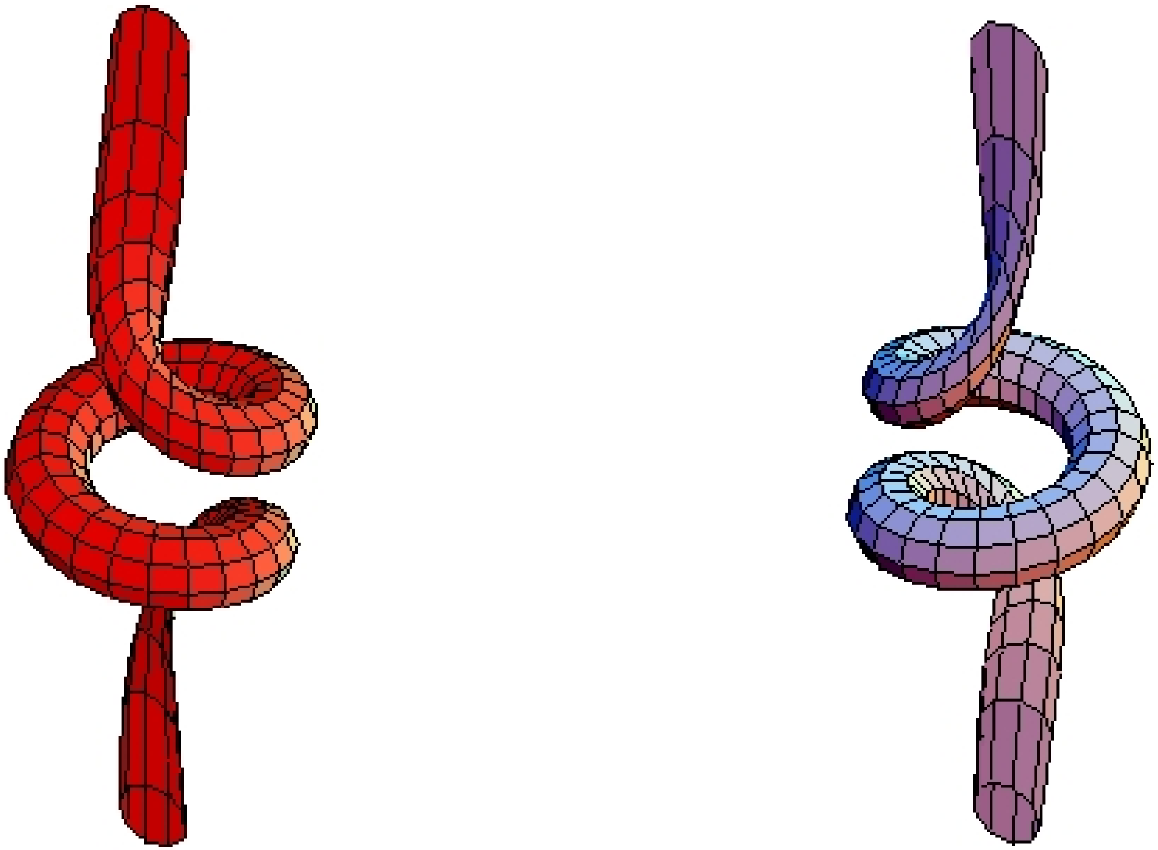
$$p = \underline{\alpha} \cdot E$$

$$p = \xi B$$

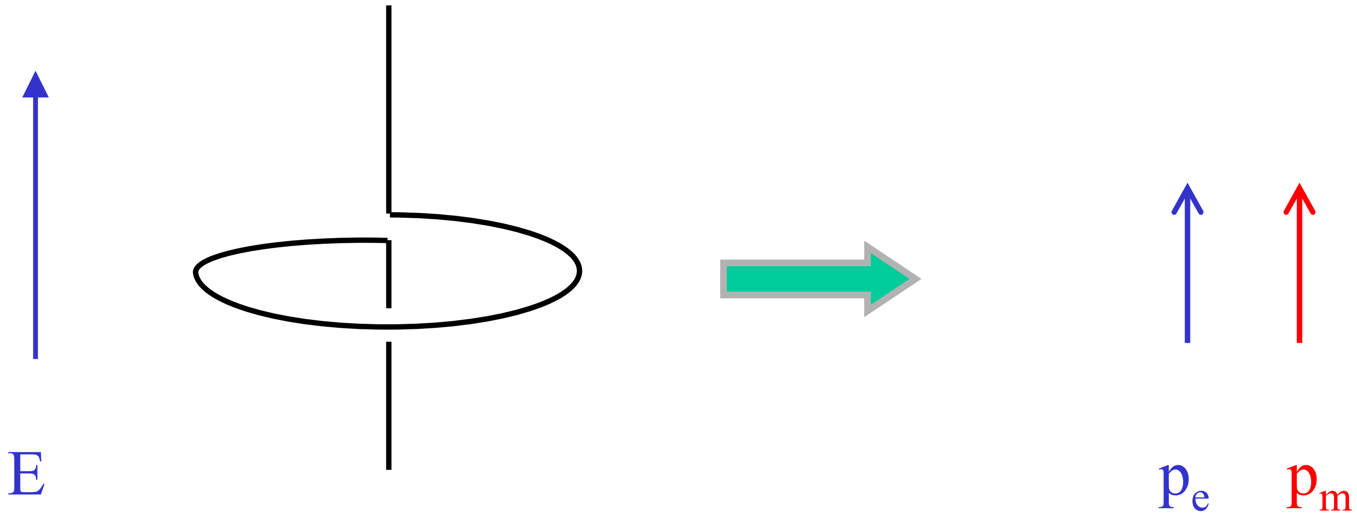
$$p = \underline{\gamma} \cdot \frac{\partial}{\partial t} E$$

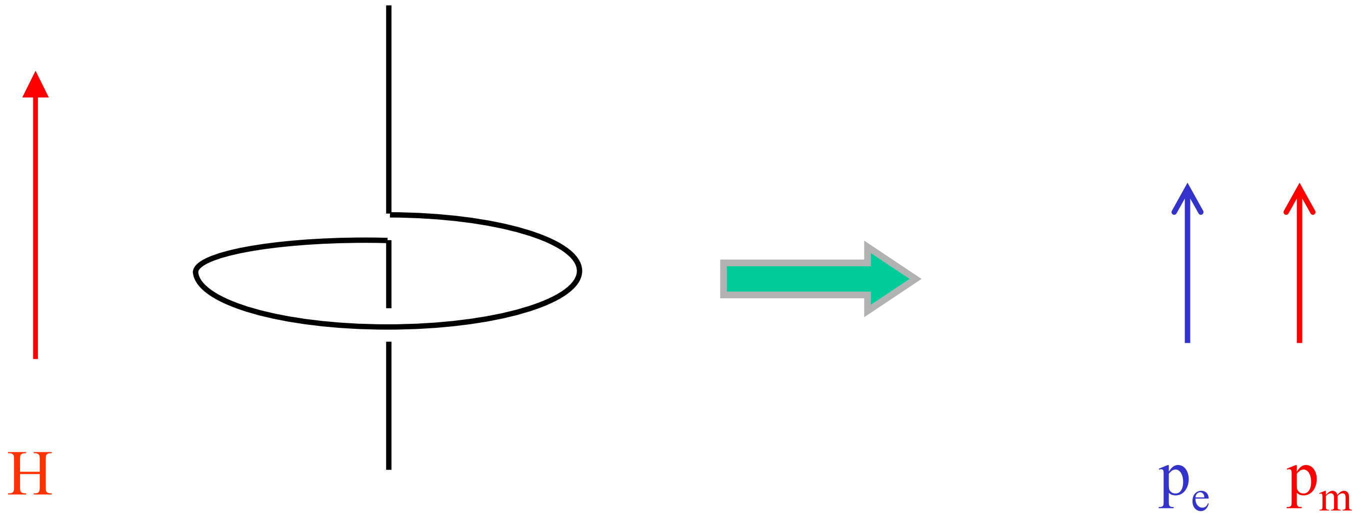
$$p = \underline{\underline{\zeta}} : \sigma$$

$$p = \underline{\underline{\beta}} : \nabla E$$

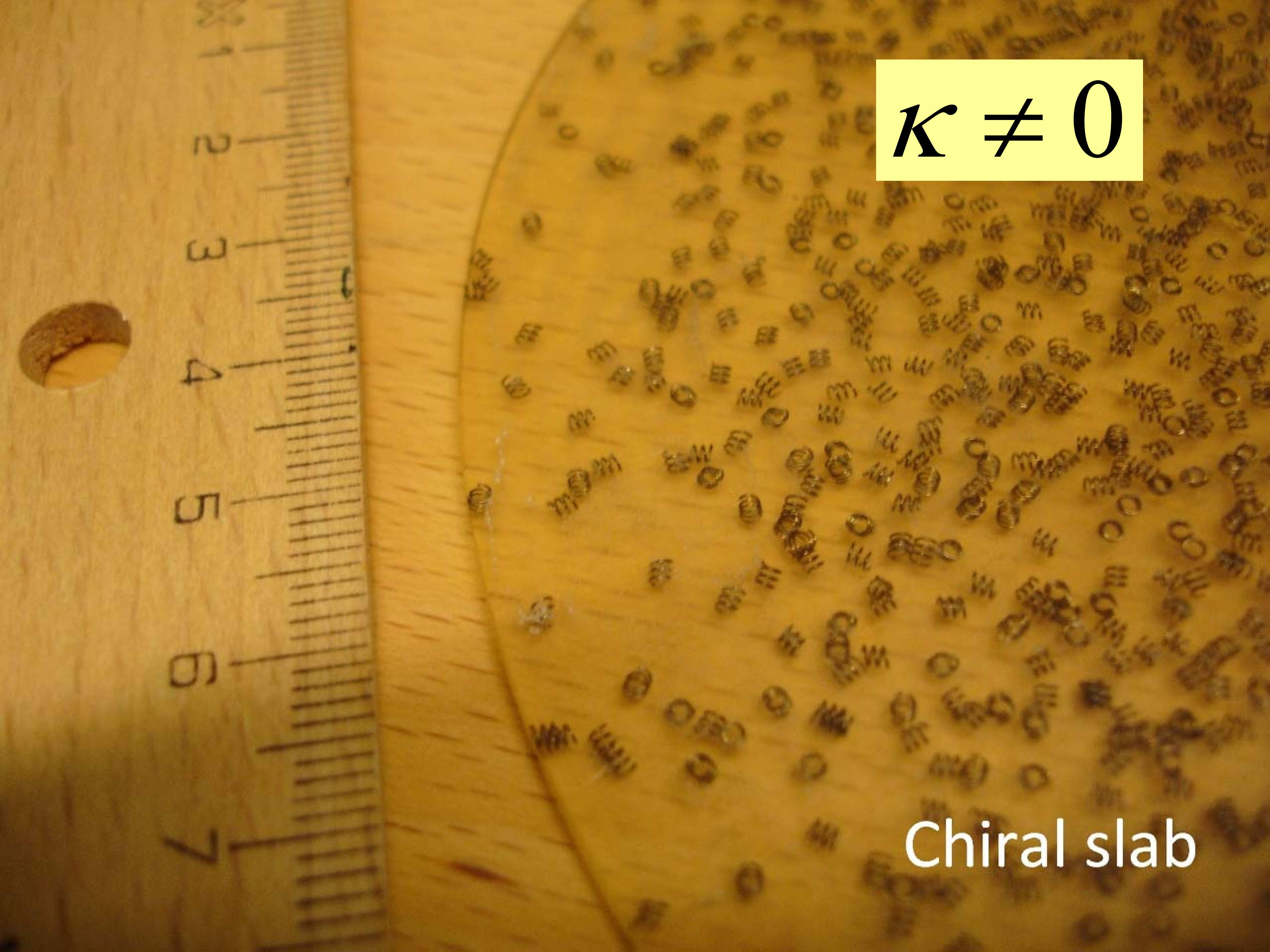


Optical activity from geometry


E
p_e
p_m



H
p_e
p_m



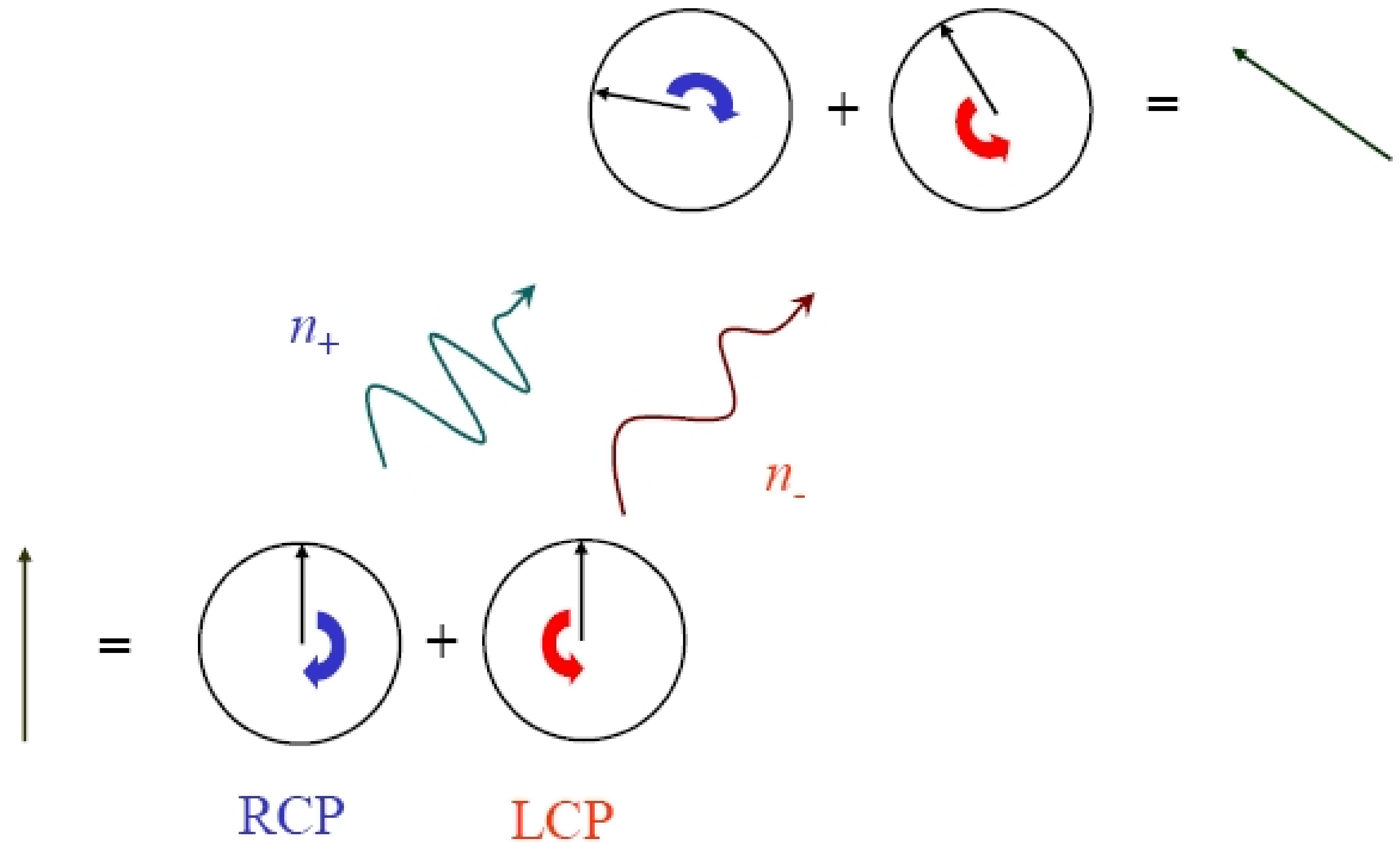
$\kappa \neq 0$
Chiral slab
1
2
3
4
5
6
7

Chiral (handed) media

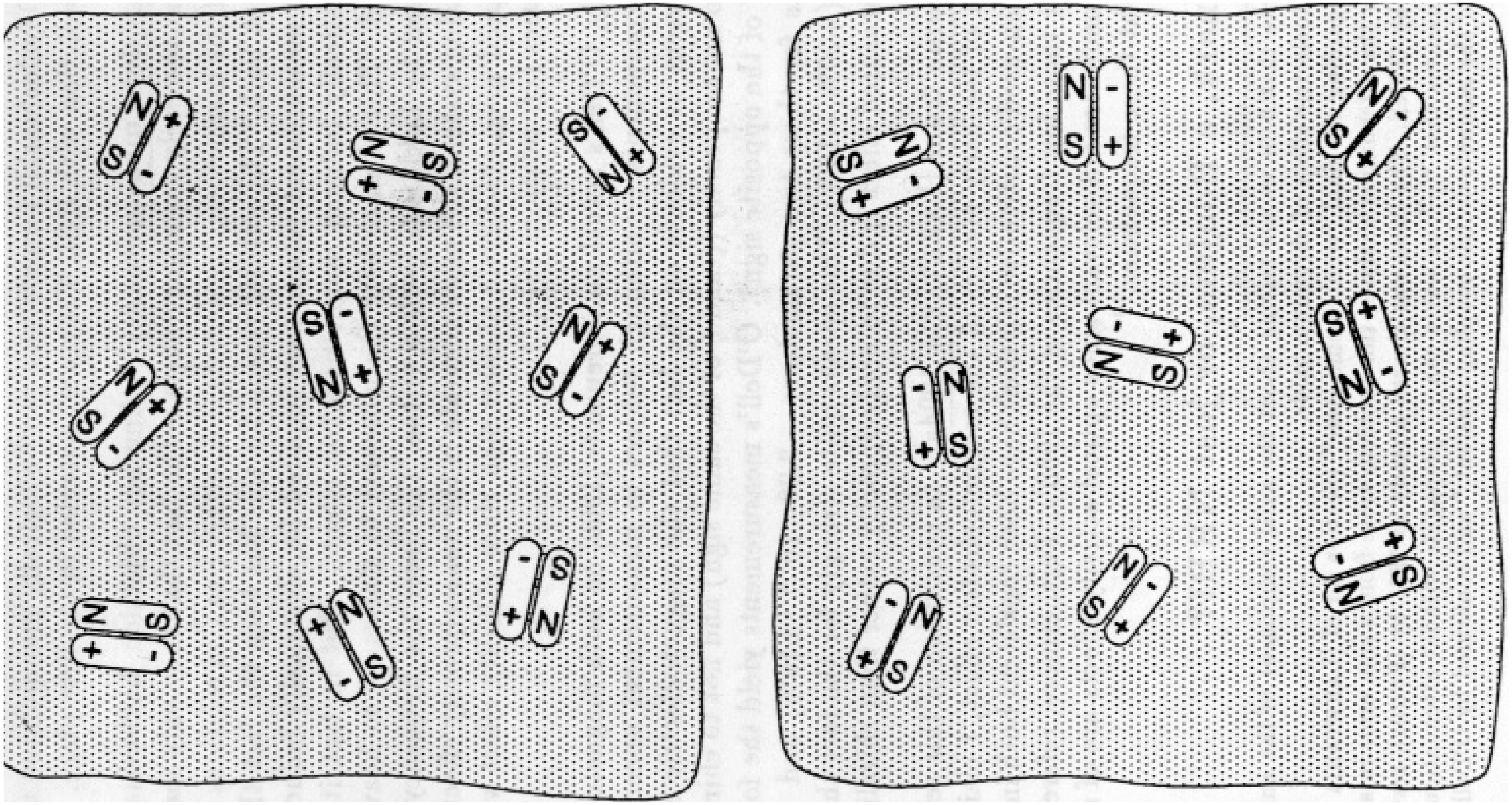
$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & -\mathrm{j}\kappa \\ \mathrm{j}\kappa & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

κ chirality parameter (Pasteur parameter)

Plane wave propagation in chiral (bi-isotropic) medium:



Tellegen (NRBI) material



Constitutive relations: bi-isotropic magnetoelectric media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

$$\xi = \chi - j\kappa$$

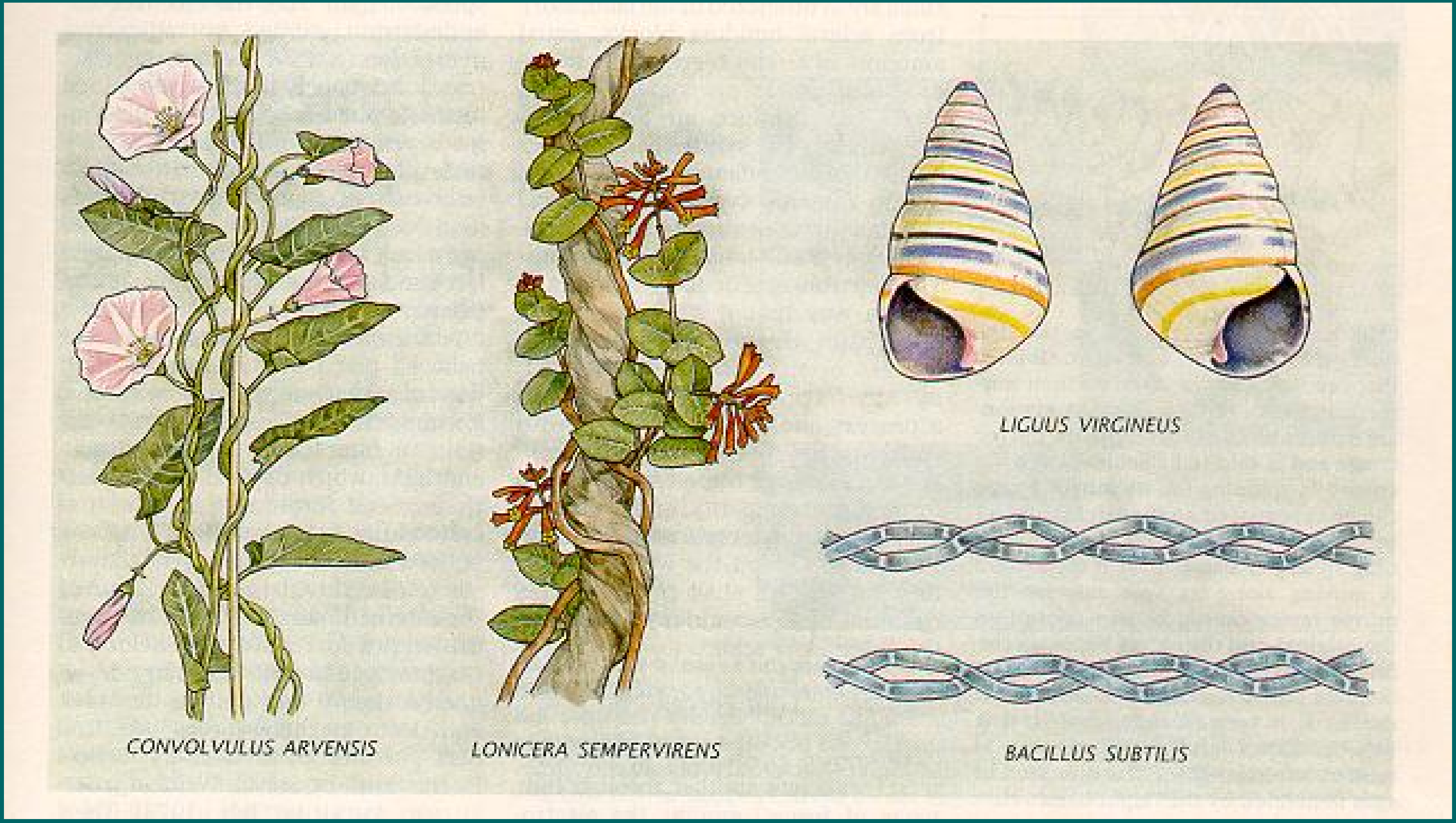
$$\zeta = \chi + j\kappa$$

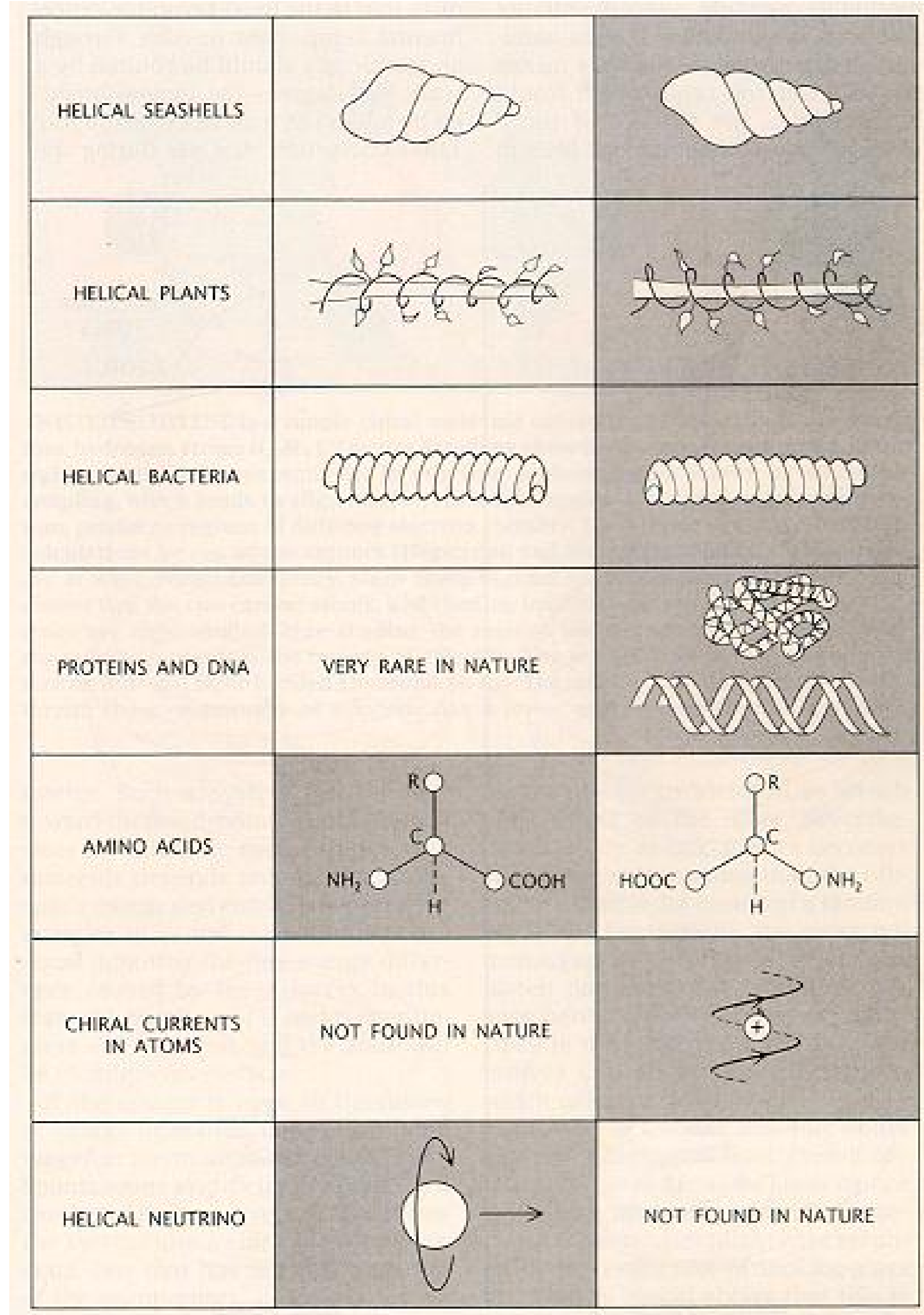
κ chirality parameter (Pasteur)

χ non-reciprocity parameter (Tellegen)

HANDEDNESS IN MATTER:

geometry and structure





Hegstrom & Kondepundi

***Scientific American*, Jan. 1990**

Australian trumpet shell

Marlinspike

Giant frog shell

Busycon perversum

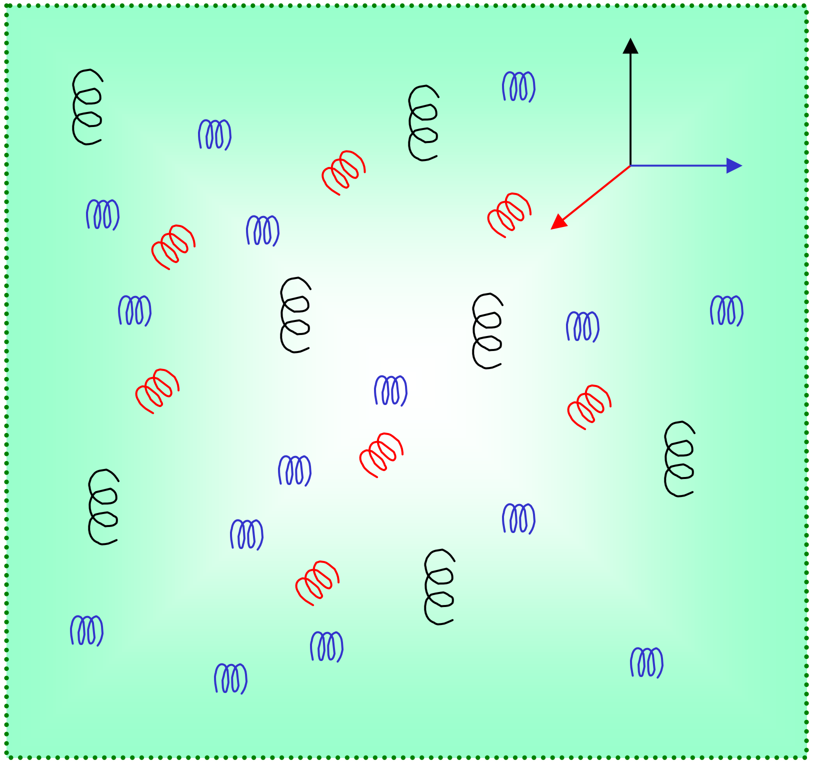
ISOTROPIC
MEDIA
ε, μ (2)

ANISOTROPIC
MEDIA
$\underline{\varepsilon}, \underline{\mu}$ (18)

BI-ISOTROPIC
MEDIA
$\varepsilon, \mu, \xi, \zeta$ (4)

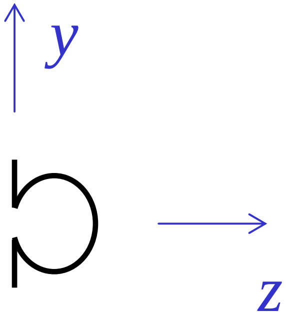
BIANISOTROPIC
MEDIA
$\underline{\varepsilon}, \underline{\mu}, \underline{\xi}, \underline{\zeta}$ (36)

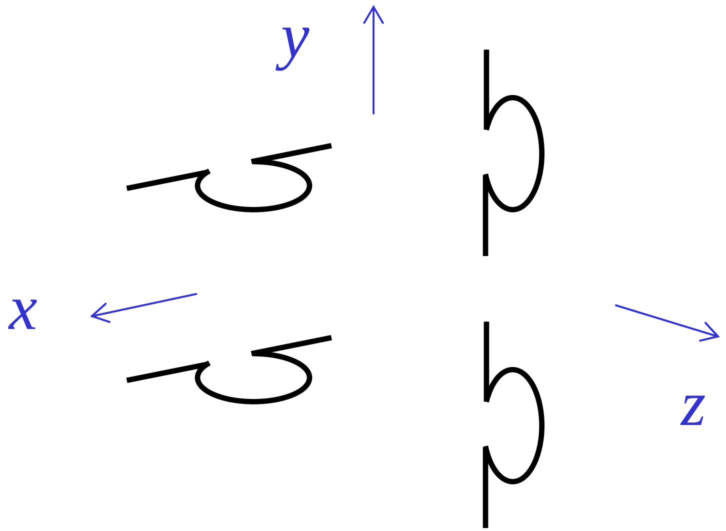
Chirality dyadic (*symmetric*)



$\kappa\,\bar{u}\bar{u} + \kappa\,\bar{u}\bar{u} + \kappa\,\bar{u}\bar{u}$

Omega medium





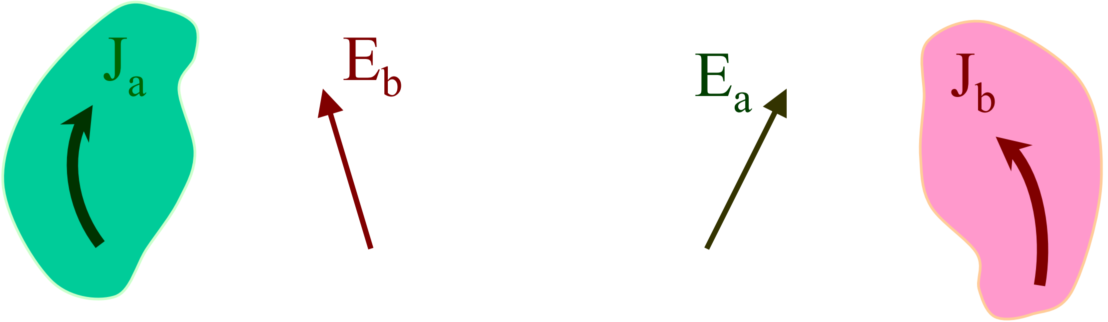
$$\xi = j\omega \begin{pmatrix} 0 & 0 & 0 \\ \Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xi = j\omega \begin{pmatrix} 0 & -\Omega & 0 \\ +\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Classification of bi-anisotropic materials

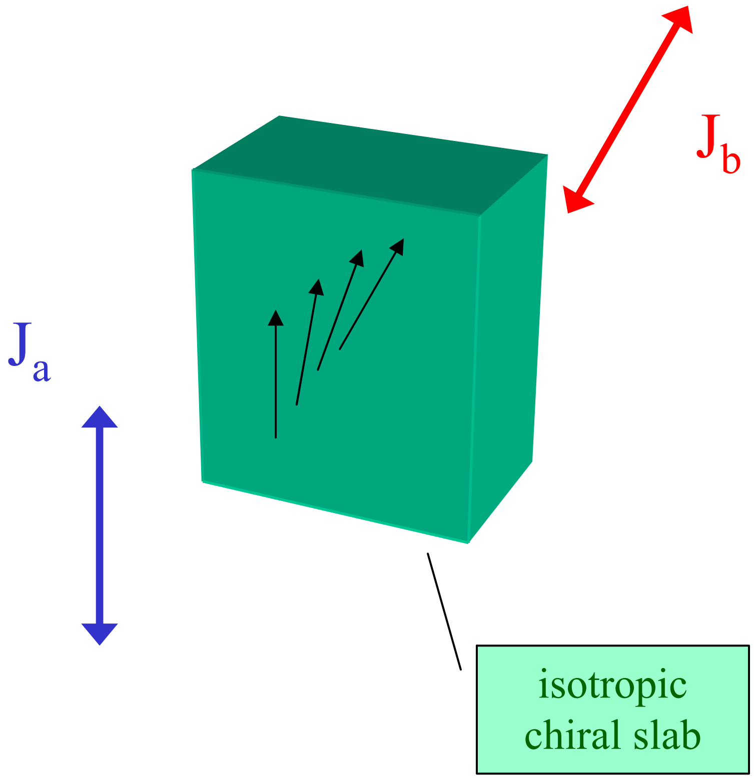
	ε	μ	κ	χ
Symmetric part: 6 parameters	(RECIPROCAL) **Dielectric crystal**	**Magnetic medium**	**Chiral medium**	**Cr_2O_3**
Anti-symmetric part 3 parameters	(NON-RECIPROCAL) **Magneto-plasma**	**Biased ferrite**	**Omega medium**	**Moving medium**

RECIPROCITY ?



$$\int \mathrm{E}_a \cdot \mathrm{J}_b \, dV \overset{?}{=} \int \mathrm{E}_b \cdot \mathrm{J}_a \, dV$$

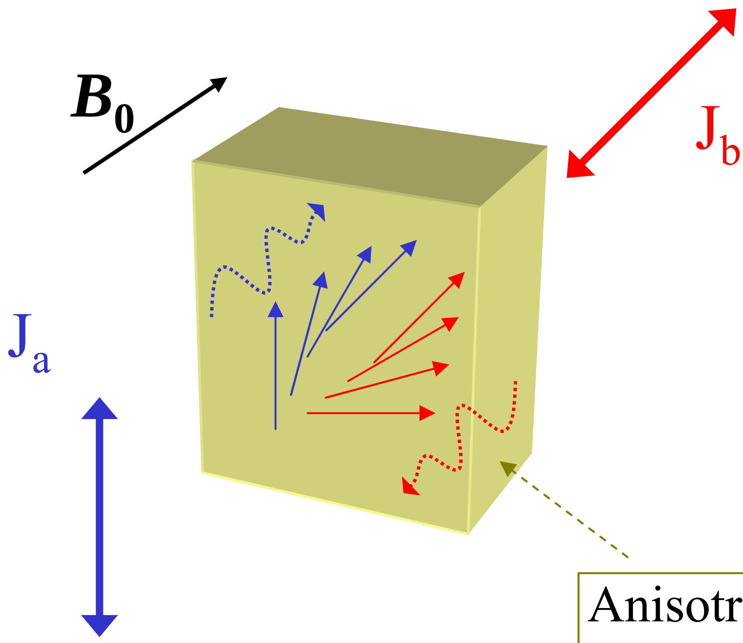
Optical activity



Pasteur medium reciprocal :

$$\int \mathrm{E}_a \cdot \mathrm{J}_b \, dV = \int \mathrm{E}_b \cdot \mathrm{J}_a \, dV$$

Faraday rotation



Magnetoplasma non - reciprocal :

$$\int E_a \cdot J_b \, dV \neq \int E_b \cdot J_a \, dV$$

Anisotropic permittivity :

$$\varepsilon = \varepsilon_{symm} + j\, g \times I$$

Kramers-Kronig relations

$$\varepsilon(\omega) = 1 + \chi(\omega); \quad \chi = \chi' - \mathrm{j}\chi''$$

$$\begin{cases} \chi'(\omega) = \dfrac{2}{\pi} \mathrm{PV} \displaystyle\int_0^\infty \dfrac{\omega' \chi''(\omega')}{\omega'^2 - \omega^2} \mathrm{d}\omega' \\ \chi''(\omega) = -\dfrac{2\omega}{\pi} \mathrm{PV} \displaystyle\int_0^\infty \dfrac{\chi'(\omega')}{\omega'^2 - \omega^2} \mathrm{d}\omega' \end{cases}$$

Global character...

losses & dispersion can separate

```
In[18]:= imaginaryPart[w_] = If[1 < w ≤ 2, 1, 0];
         desimaalit = 50;
         w0 = SetPrecision[0.1, 2 desimaalit];
         w1 = SetPrecision[10, 2 desimaalit];
         lkm = 200;
         eInfinity = 1;
         realPart[w_] :=
           eInfinity + 2/π CauchyPrincipalValue[imaginaryPart[ww] ww/(ww^2 - w^2), {ww, 0, {w}, Infinity},
             WorkingPrecision → desimaalit, PrecisionGoal → 4];
```

```
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```
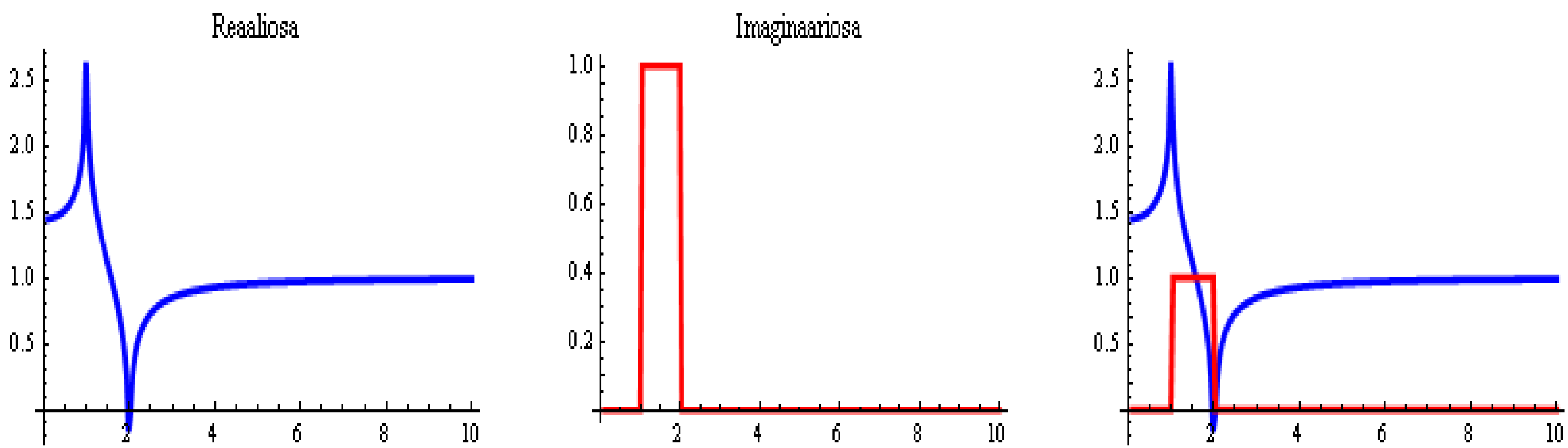



High losses

→ **Dispersion!**

Dielectric models for dispersion

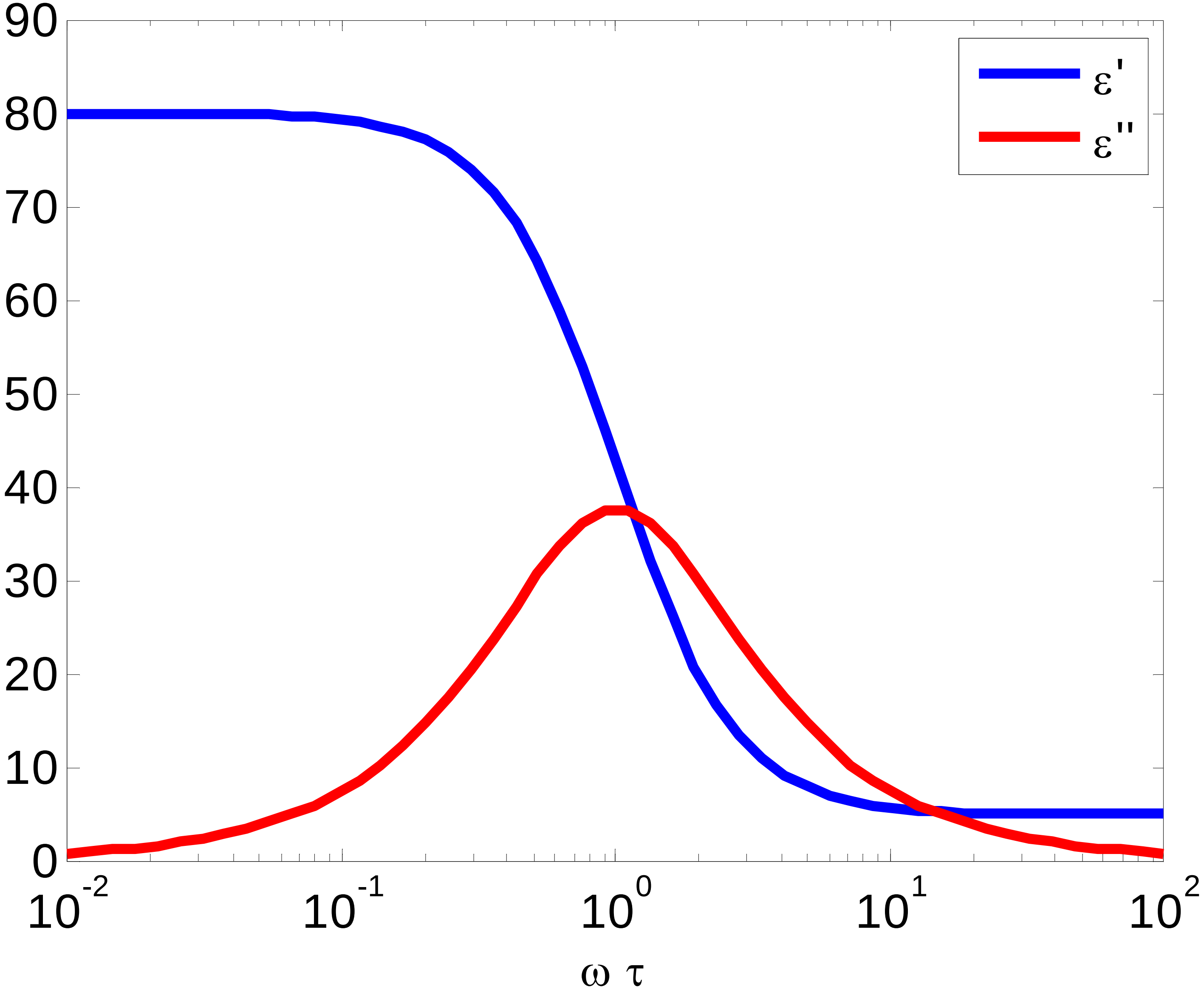
Debye model

$$\varepsilon(\omega) = \varepsilon'(\omega) - \mathrm{j}\varepsilon''(\omega)$$

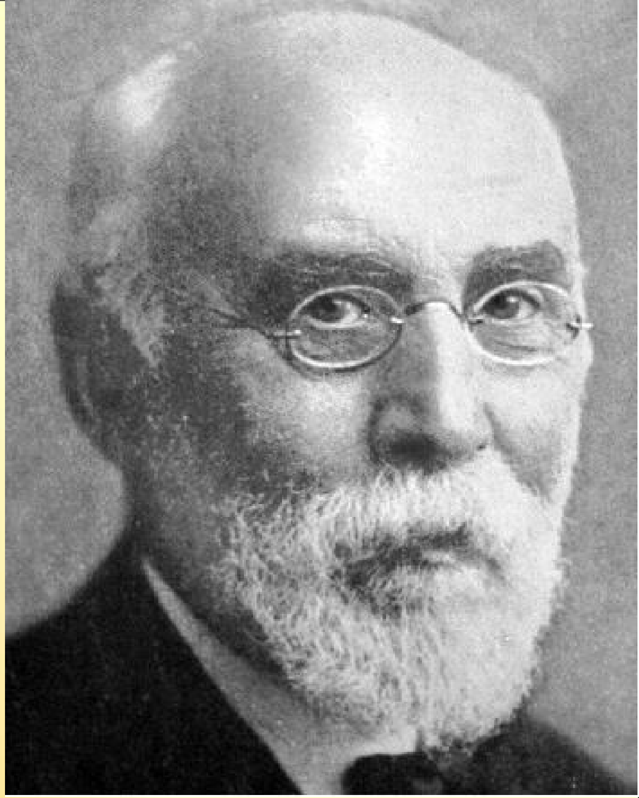
$$= \varepsilon_\infty + \frac{\varepsilon_\mathrm{s} - \varepsilon_\infty}{1 + \mathrm{j}\omega\tau}$$

Debye model

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}$$



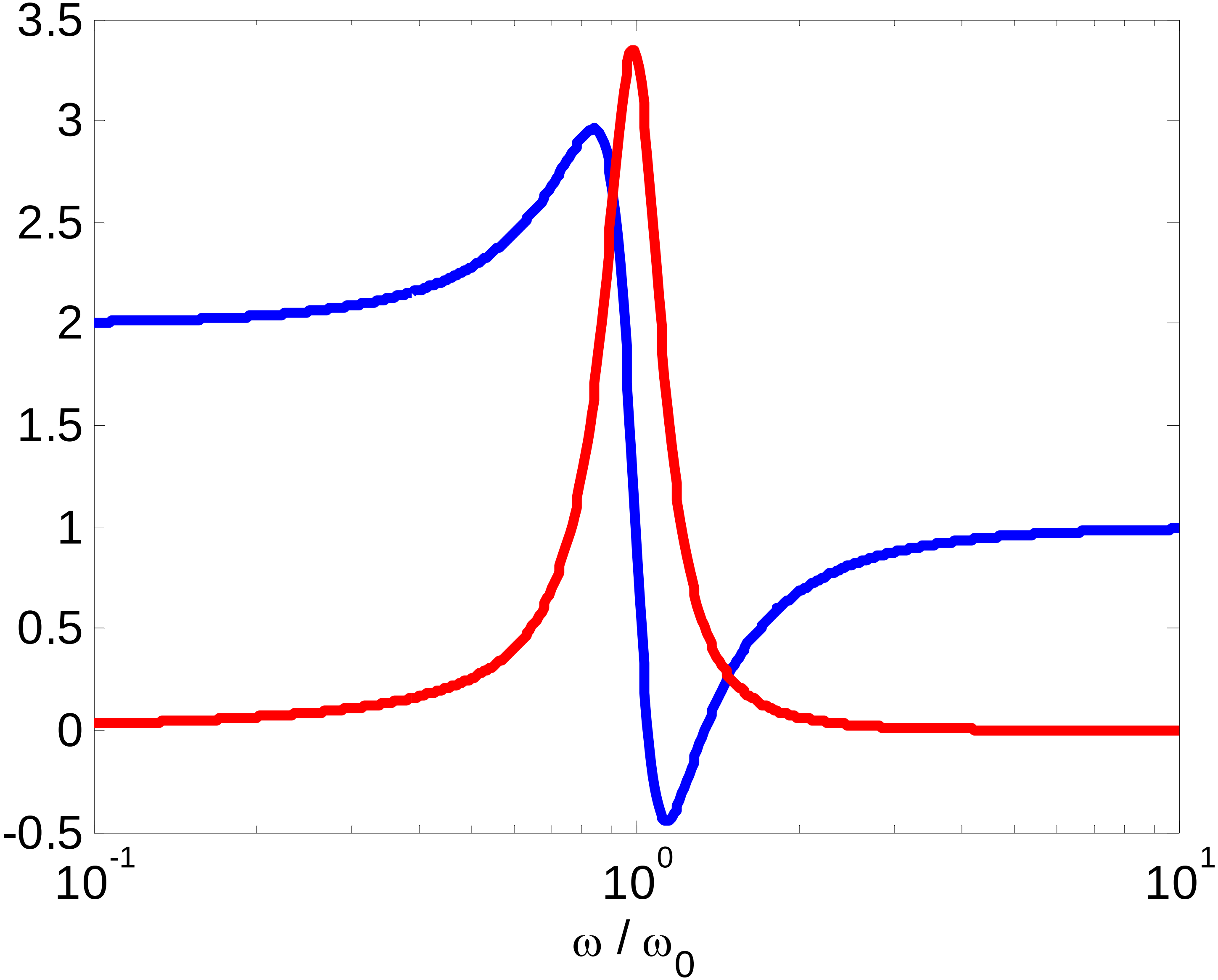
Lorentz model



$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + j\nu\omega}$$

Lorentz model

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + j\nu\omega}$$

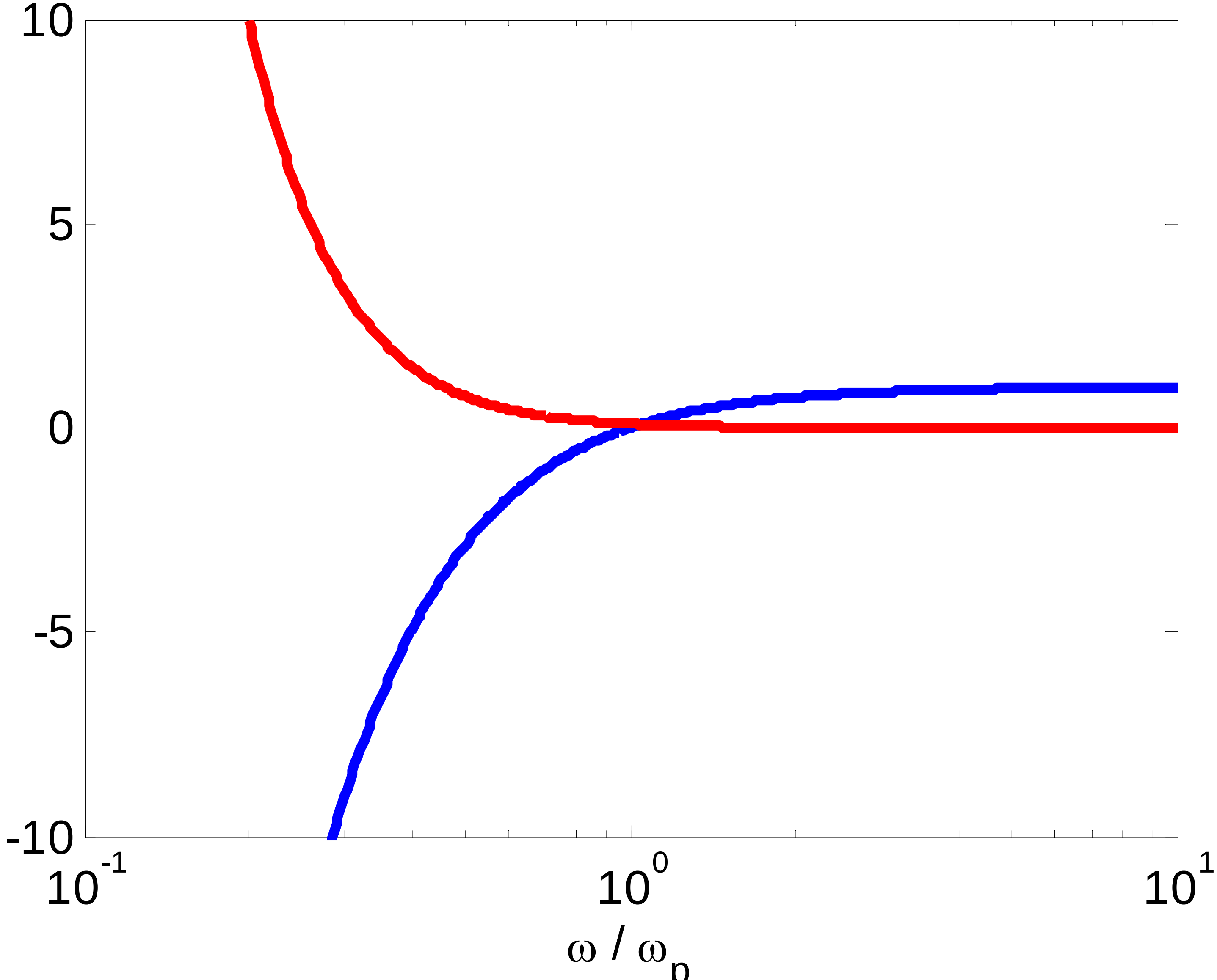


Drude model

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_{\mathrm{p}}^{2}}{\omega^{2} - \mathrm{j}\nu\omega}$$

Drude model

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_{\mathrm{p}}^2}{\omega^2 - \mathrm{j}\nu\omega}$$

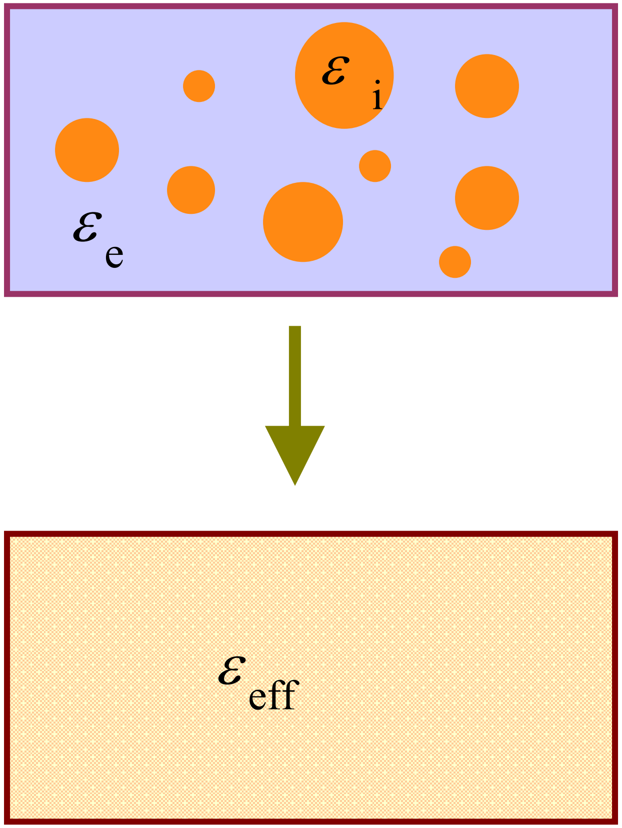


Heterogeneities

- Modeling
- New effects

Maxwell Garnett mixing formula

$$\varepsilon_{\mathrm{eff}} = \varepsilon_{\mathrm{e}} + 3 f \varepsilon_{\mathrm{e}} \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{e}} - f(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}$$



Maxwell Garnett

$$\frac{\varepsilon_{\mathrm{eff}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{eff}} + 2\varepsilon_{\mathrm{e}}} = f \, \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{e}}}$$

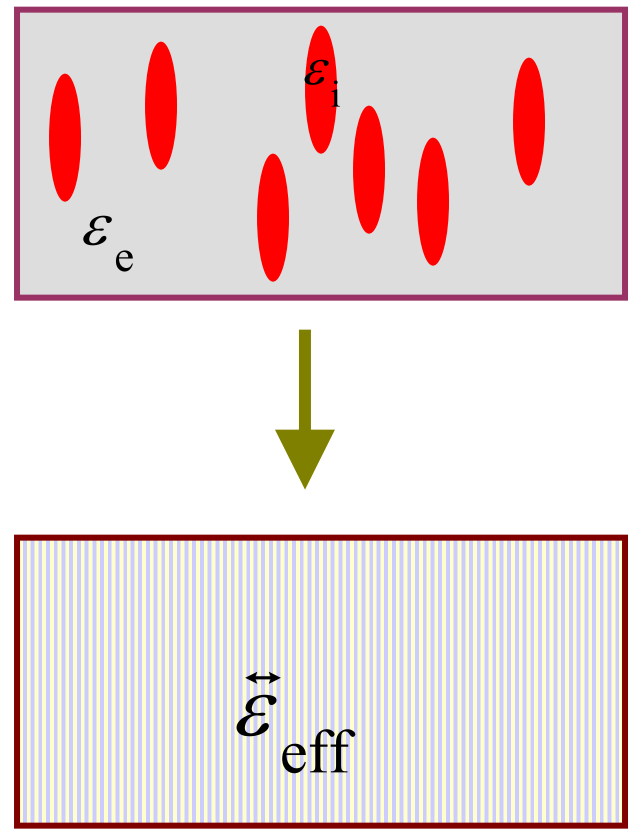
Bruggeman (symmetric)

$$(1 - f)\frac{\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{eff}}}{\varepsilon_{\mathrm{e}} + 2\varepsilon_{\mathrm{eff}}} + f \, \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{eff}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{eff}}} = 0$$

Aligned ellipsoids

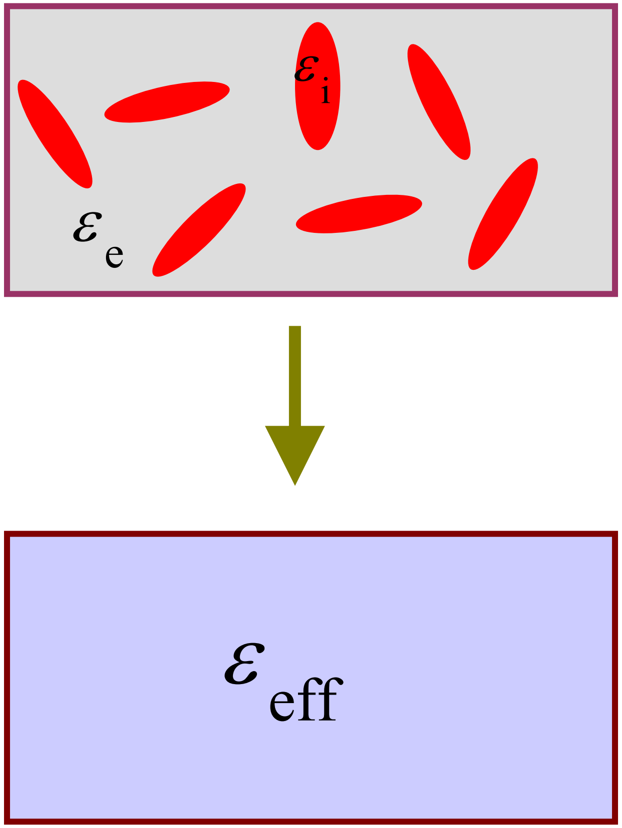
$$\varepsilon_{\mathrm{eff},x} = \varepsilon_{\mathrm{e}} + f\,\varepsilon_{\mathrm{e}} \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{e}} + (1-f)N_x(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}$$

depolarization factor !



Randomly oriented ellipsoids

$$\varepsilon_{\mathrm{eff}} = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{e}} \frac{\dfrac{f}{3} \displaystyle\sum_{j=x,y,z} \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{e}} + N_j(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}}{\varepsilon_{\mathrm{e}} - \dfrac{f}{3} \displaystyle\sum_{j=x,y,z} \frac{N_j(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}{\varepsilon_{\mathrm{e}} + N_j(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}}$$



(isotropic!)