Additional explanations on the experimental characterization road map (for experts in the field)

Step 1

The "road map" of the previous section concerns electromagnetic characterization of artificial heterogeneous media, in which the interaction of electromagnetic fields with constitutive elements is of resonant nature or, more generally, the solenoidal part of the fields plays significant role in this interaction. This can be the case of metamaterials or photonic crystals or something else, and we cannot determine that without a careful experimental examination.

First of all, we should find out what kinds of physical effects determine the response of our system to electromagnetic excitations. At this point we have the first fork in our research journey. Due to scattering by separate inclusions, the incident coherent wave partly changes into new coherent waves and partly to incoherent diffusion radiation. Several coherent plane waves can be created if we deal with a photonic crystal (the period comparable to or larger than the wavelength, the Borrmann effect). The emergence of a single coherent wave in reflected field is dictated by the material Maxwell equations with some kind of constitutive relations, whereas the emergence and transport of the incoherent part is described by diffusion equations. The radiation transfer is quite different in these regimes. For example, light differently travels through a crystal of sugar and through granulated sugar. Thus, we have to experimentally determine the intensity of diffusion component in scattering as well as possible presence of grating lobes. It is useful to consider oblique incidence of light, because in this case an effective scale of inhomogeneity increases and may become comparable to the wavelength. One can observe some anomalous behavior like Wood anomalies. Note that changing the sample size or the frequency we can pass from coherent reflection and scattering to diffusion one. Indeed, weak diffusion scattering can be described in terms of the imaginary part of permittivity $\epsilon''_{\rm eff}$ [6], which describes instead of dissipation the transformation of coherent radiation into diffusion one. It is obvious that at distances greater than $L = \lambda/\sqrt{k_0\epsilon''_{\rm eff}}$ most part of the energy is carried by diffusion component. If we change the frequency, sooner or latter the effective wavelength inside the material sample becomes equal to the characteristic size of inhomogeneities or to the distances between them. In this case the scattering may become diffusive. If the level of diffusion scattering is small, we can pass to Step 2.

Step 2

It is important to keep in mind that electromagnetic scattering cross-section of a resonant dipole particle is much greater than its geometrical cross-section, namely, the cross-section is proportional to square of the wavelength λ. Thus, we can expect that the wave "feels" this dipole particle at distance about λ. Such a nonlocal interaction can result in many unusual effects. The properties of the material may change not only with the sample shape but also for different surrounding bodies, because the vacuum in the vicinity of a spatially dispersive material sample also becomes a "spatially dispersive medium" [7]. Thus, it is reasonable to test the sample by cladding it with usual materials (a metal screen or a dielectric sample with a high value of permittivity). The evanescent waves, extending from the sample could transform into travelling waves interacting with metal films (plasmon generation) or getting into a high-permittivity medium.

To evaluate the influence of the sample shape it reasonable to place the sample on a diffraction grating. The period of the grating may serve as a yard scale of the geometrical shape. If there are no additional waves, we can pass to Steps 3 and 4: Retrieval of local constitutive parameters.

Steps 3 and 4

Usually, plane-wave probing is used at these stages, but in case of negative constitutive parameters it is possible to employ plasmonic techniques. Here, the sample is brought close to a sample with the known positive parameters, and excitation of surface plasmons is studied. The main purpose is to determine a minimal set of constitutive parameters we need and measure them. The set should be in agreement with the symmetry properties of the sample as a whole and of individual inclusions.

Step 5

Note that we have to study the frequency dispersion of the retrieval parameters in order to evaluate if the parameters satisfy the Kramers-Kronig relations. If at least one of the above criteria which we check at Step 5 is not satisfied, this means that the adopted effective parameter model and / or retrieval process are not adequate for this structure and need to be improved. Usually this means a more complicated model (more “effective material parameters”). In particular, if the so called “anti-resonance” is observed (one of the material parameters shows a non-physical resonance with the wrong sign of the imaginary part and/or non-physical frequency dispersion of the real part), it can be possible to modify the retrieval procedure taking into account a surface non-locality of the material (jump of macroscopic fields across its surface). There are also alternative approaches to the solution of this problem under development in several laboratories, but no complete theory and no fully developed experimental approach exists at this stage.

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