Split rings and wire media

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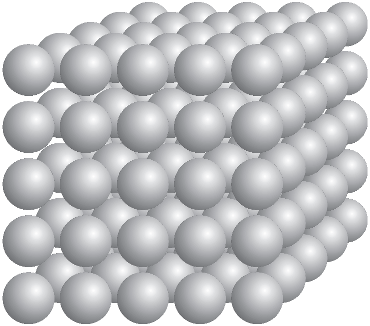
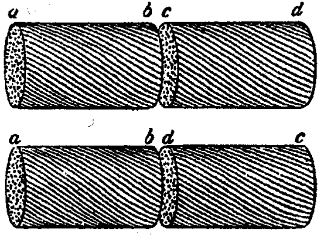
Lecture plan

- Split rings
  - History and basic properties
  - Circuit model
  - Lorentz dispersion
- Wire media
  - History and basic properties
  - Circuit model
  - Drude dispersion
  - Strong spatial dispersion

Imitating nature. . .



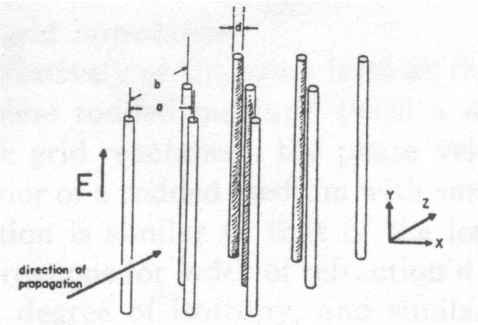


Artificial dielectrics

Artificial chiral materials

The right figure from J.C. Bose, On the rotation of plane of polarization of electric waves by twisted structure, *Proc. Royal Soc.*, vol. 63, pp. 146-152, 1898.

First "original metamaterial designs"



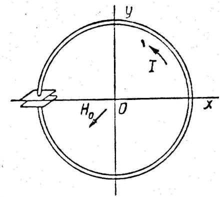
$$\epsilon_p = \epsilon_0 \left(1 - \frac{{\omega_p}^2}{\nu^2 + \omega^2} + j\,\frac{{\omega_p}^2 \nu/\omega}{\nu^2 + \omega^2}\right)$$

J. Brown, 1953; W. Rotman, 1961; J. Pendry, 1996

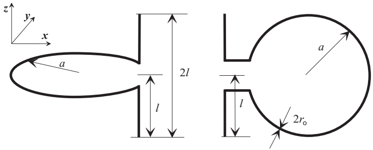
First DNG/Veselago material

R.A. Shelby, et al., *Science*, vol. 292, pp. 77-79, 2001.

Split rings

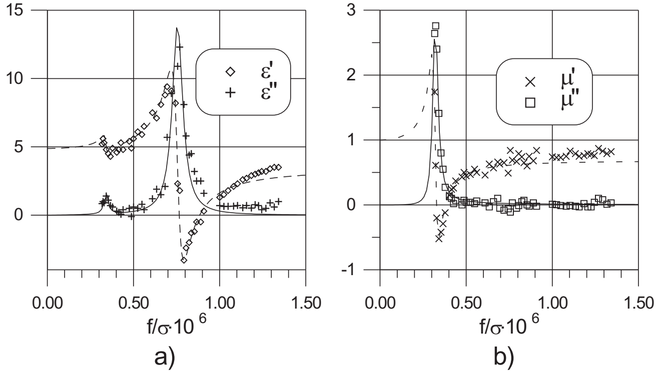


$$\chi_m^0 = \frac{\omega^2\mu_0^2 C S^2}{1-\omega^2 LC}$$



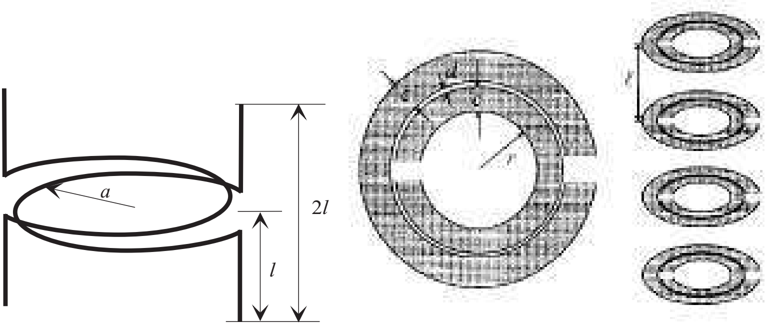
S.A. Schelkunoff, H.T. Friis, 1952; D. Jaggard, N. Engheta, and many other authors, 1980–1990

$\mu < 0$, first(?) experiment



A.N. Lagarkov, V.N. Semenenko, V.A. Chistyaev, D.E. Ryabov, S.A. Tretyakov, C.R. Simovski, Resonance properties of bi-helix media at microwaves, *Electromagnetics,* vol. 17, no. 3, pp. 213-237, 1997.

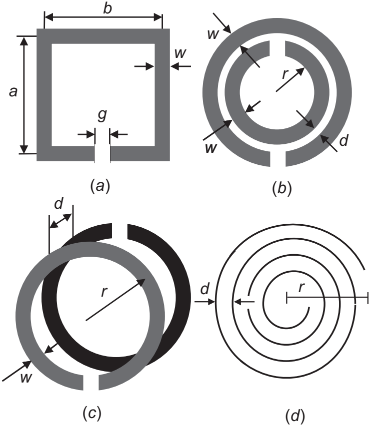
Negative permeability, SRR

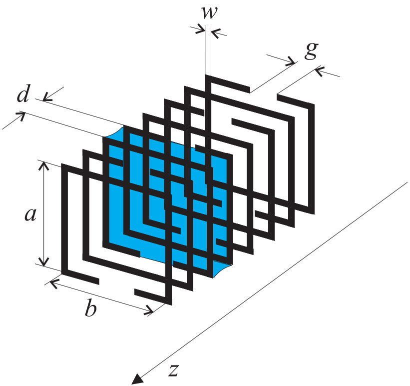


A.N. Lagarkov, et al., *Electromagnetics,* vol. 17, no. 3, pp. 213-237, 1997 (left); J.B. Pendry, et al., *IEEE Trans. Microwave Theory Techn.*, vol. 47, pp. 2075-2084, 1999 (right).

Realization of negative permeability

Variations of split-ring resonators





Artificial magnetics.

Basic principles

- You need to create loop currents ⇒ make loop-shaped conductors

$$I = \frac{-j\omega SB}{j\omega L} = -\frac{SB}{L}$$

- You want to control the phase of the induced magnetic moment ⇒ load the loops with capacitors

$$I = \frac{-j\omega SB}{j\omega L + \frac{1}{j\omega C}} = \frac{\omega^2 SB}{1 - \omega^2 LC}$$

Low-frequency limit $\omega \ll \omega_0$: $I \approx \omega^2 CSB$, $m \approx \omega^2 \mu_0 CS^2 B$ — OK.

High-frequency limit $\omega \to \infty$: $m \to -\mu_0 \frac{S^2 B}{L}$ — non-physical.

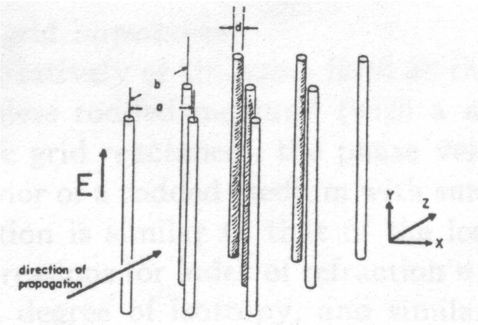
The operational principle

Simple and the same for all these particles:

$$I = \frac{\mathcal{E}^{\mathrm{ext}}}{Z_{\mathrm{tot}}} = \frac{\mathcal{E}^{\mathrm{ext}}}{j\omega L_{\mathrm{eff}} + \dfrac{1}{j\omega C_{\mathrm{eff}}} + R_{\mathrm{eff}}}$$

$$m = \mu_0 NSI = \frac{\omega^2 \mu_0^2 N^2 S^2 C_{\mathrm{eff}} H^{\mathrm{ext}}}{1 - \omega^2 L_{\mathrm{eff}} C_{\mathrm{eff}} + j\omega R_{\mathrm{eff}} C_{\mathrm{eff}}}$$

Wire media: History



$$\epsilon_p = \epsilon_0 \left(1 - \frac{{\omega_p}^2}{\nu^2 + \omega^2} + j \frac{{\omega_p}^2 \nu / \omega}{\nu^2 + \omega^2} \right)$$

J. Brown, 1953; W. Rotman, 1961; J. Pendry, 1996

Wire medium

More than just negative permittivity:

$$\epsilon_{zz}(\omega, k_z) = \epsilon_0 \left(1 - \frac{k_p^2}{k^2 - k_z^2}\right)$$

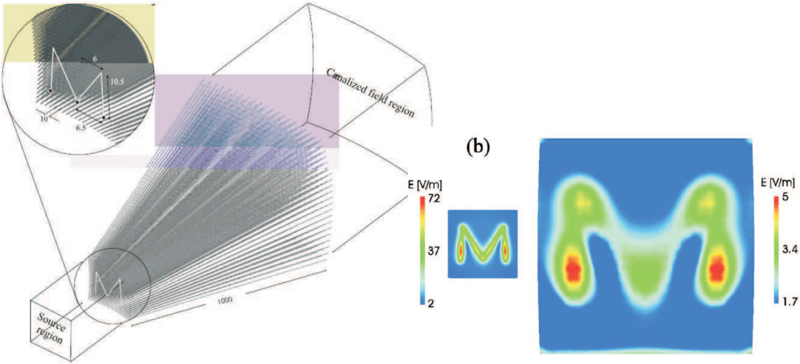
In space-time domain:

$$\mathbf{D}(x,y,z) = \epsilon_0 \mathbf{E}(x,y,z) + \frac{\epsilon_0 k_p^2 c}{4} \mathbf{z}_0 \int_{-\infty}^{t} \int_{z-c(t-t')}^{z+c(t-t')} E_z(x,y,z',t')\, dz'\, dt'$$

G. Shvets, Advanced Accelerator Concepts: Tenth Workshop, edited by C. E. Clayton and P. Muggli, 2002; P.A. Belov, et al., *Phys. Rev. B,* vol. 67, 113103, 2003.

Canalization of waves in wire media

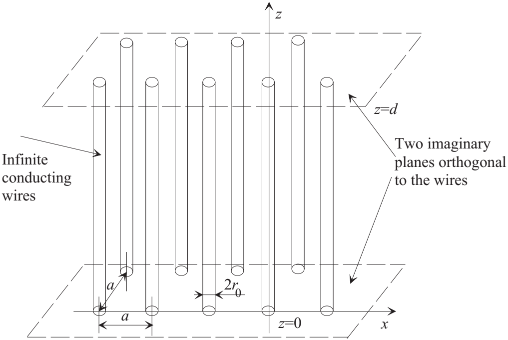
Enlarging superlens



P. Ikonen, C. Simovski, S. Tretyakov, P. Belov, and Y. Hao, *Applied Physics Lett.*, vol. 91, p. 104102, 2007.

Quasistatic model of wire media

S.I. Maslovski, S.A. Tretyakov, P.A. Belov, Wire media with negative effective permittivity: a quasi-static model, *Microwave and Optical Technology Lett.*, vol. 35, no. 1, pp. 47-51, 2002.



$$U = Ij\omega Ld \quad \text{and} \quad U = E_z d \quad \Rightarrow \quad E_z = j\omega L\, I$$

Quasistatic model of wire media

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \text{where} \quad \mathbf{P} = \frac{\mathbf{J}}{j\omega} = \mathbf{z}_0 \frac{I}{j\omega\, a^2} = -\mathbf{z}_0 \frac{E_z}{\omega^2\, a^2\, L}$$

Material relation:

$$D_z = \left(\epsilon_0 - \frac{1}{\omega^2\, a^2\, L} \right) E_z$$

Inductance per unit length L

Approximation:

$$H_y = \frac{I}{2\pi}\left(\frac{1}{x} - \frac{1}{a-x}\right)$$

Magnetic flux per unit length:

$$\Psi = \mu_0 \int_{r_0}^{a/2} H_y(x)\,dx = \frac{\mu_0 I}{2\pi} \log \frac{a^2}{4r_0(a-r_0)}$$

Inductance per unit length:

$$L = \frac{\mu_0}{2\pi} \log \frac{a^2}{4r_0(a-r_0)}$$